

Comment on “Study of Excess Conduction Loss in Normal Metals at/below Sub-millimeter Wavelengths” by W. Pan and X. Zhang, *Int. J. of Infrared and Millimeter Waves*, vol. 27, no. 3, 455–464, Mar. 2006

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1 Introduction

The recent paper by Pan and Zhang [1], and the one that precedes this [2], represent work undertaken by Professor Zhang while on a 5-month sabbatical at Imperial College London. However, at no time has the model within these papers been endorsed for publication by anyone at Imperial College London. Unfortunately, there are a number of basic fundamental flaws in their work and these will be discussed here in detail.

2 Pan and Zhang model

The author of this comment has previously investigated modeling strategies for normal metals. In one study, experimental measurements that suggested the possibility of anomalous room-temperature conduction losses were examined between DC and 12.5 THz [3]. It was found that the classical relaxation-effect model was still valid up to these frequencies. In another study, an elaborate semiclassical model to describe anomalous excess conduction losses at room temperature was found to be completely erroneous [4]. In order to create accurate analytical models, it is important to develop semiclassical modeling strategies [5] or develop quantum mechanical treatments. Pan and Zhang use a quantum mechanical approach to derive, in detail [2], the following expression for the dielectric function of the free-electron gas [1,2]:

$$\epsilon_r(q, \omega) = 1 + \frac{2e^2}{q^2 \epsilon_0} \frac{1}{V} \sum_k \frac{f(k) - f(k+q)}{E(k+q) - E(k) - \hbar\omega + j\zeta} \quad (1)$$

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where, e =electron charge; $f(k)$ =Fermi-Dirac distribution function; k =wave vector of the electron; modified wave number of the driving electric field $q = q' - jq''$; ϵ_o = permittivity of free space; V =volume; angular frequency; $\hbar$ =modified Planck's constant; angular frequency, $\omega = 2\pi f$; and f =frequency of the driving electric field.

This is a standard derivation and the result is known as the Lindhard dielectric function; also known as the self-consistent-field (SCF) or random phase approximation (RPA) dielectric function [6]. By replacing $f(k+q)$ with $f(-k)$ and taking the long wavelength limit, Pan and Zhang highlight the following [1,2]:

$$\frac{1}{V} \sum_k f(k) = \frac{n}{2} \quad ; \quad \sigma_o = \frac{ne^2\tau}{m} \quad ; \quad \nu_f = \frac{\hbar k_f}{m} \quad ; \quad \zeta = \frac{\hbar}{2\tau} \quad (2)$$

where n = electron density, σ_o = intrinsic bulk conductivity at DC, τ = phenomenological scattering relaxation time for the free electrons (i.e. mean time between collisions), m = effective electron mass, ν_f = velocity between collisions of the free electron, having kinetic energy at the Fermi level and k_f = Fermi wave number; as a means to somehow derive an expression for their intrinsic bulk conductivity, in the following expressions for their dielectric function and surface impedance [1,2]:

$$\epsilon_{rZP}(q, \omega) = 1 + \frac{\sigma_{ZP}(q, \omega)}{j\omega\epsilon_o} \quad \text{where} \quad \sigma_{ZP}(q, \omega) = \frac{\sigma_o}{1 + j\omega\tau \left[1 - \frac{1}{2} \left(\frac{q l_m}{\omega\tau} \right)^2 \right]} \quad (3)$$

$$\therefore Z_{ZP} = \sqrt{\frac{j\omega\mu_o\mu_r}{\sigma_{ZP}(q, \omega) + j\omega\epsilon_o}}$$

where, l_m = mean distance traveled by the electron between collisions (i.e. mean-free path length).

Unfortunately, the original expression for the dielectric function given in (1) is only valid for longitudinal wave propagation [6]. As a result, the Pan and Zhang model has no meaning for their surface impedance, Z_{ZP} , and excess conduction loss calculations (as they are based on transverse wave propagation for normal incidence).

It is evident from the publications of Pan and Zhang that they have made the mistake of trying to fit the wrong type of theoretical model to measured data. It has been previously demonstrated that the relaxation-effect model is adequate for characterising the intrinsic dispersive nature of normal metals at room temperature, even into the terahertz frequency range [3]. Indeed, the results from a quantum mechanical model should actually converge onto those from the relaxation-effect model at these frequencies, at room temperature. Since the room temperature results, from the model of Pan and Zhang, deviate from the classical relaxation-effect model at relatively low frequencies then this alone points to a fundamental error.

3 Proof of contradictions

Apart from the lack of any physical insight to justify the need for a new model and the obvious misuse of longitudinal wave propagation terms for calculating surface impedance, the work published by Pan and Zhang have a couple of fundamental contradictions.

3.1 First contradiction

Intrinsic bulk conductivity exhibits spatial dispersion in the non-local response regime and, therefore, has a wave number dependency in reciprocal q -space. For one-dimensional

propagation along the positive z -axis, the general expression for conduction current density, J_c , in terms of intrinsic bulk conductivity and electric field, E , is given by the non-local constitutive equation in real space [7]:

$$J_{cy}(z)|_{p=1} = \int_{-\infty}^{+\infty} \sigma(z-z')E_y(z').dz' \quad (4)$$

where p =specular reflection coefficient

Now, at the surface of the metal, conduction current density vector, $\mathbf{J}_{cy}(\mathbf{0})$, and surface current density vector, $\mathbf{J}_s$, are related by the propagation constant at the surface, $\gamma(0)$, as follows:

$$\mathbf{J}_{cy}(\mathbf{0}) = \gamma(0)\mathbf{J}_s \quad \text{where} \quad \mathbf{J}_s = \mathbf{n} \times \mathbf{H}_x(\mathbf{0}) \quad (5)$$

where, $\mathbf{n}$ =unit vector pointing out normal to the surface of the metal and $\mathbf{H}_x(\mathbf{0})$ =magnetic field vector at the surface of the metal. Equation (4) can be expressed in terms of a simple convolution (denoted by the symbol $*$) of the intrinsic bulk conductivity and electric field as follows:

$$\therefore J_{cy}(z)|_{p=1} \equiv \{\sigma(z)*E_y(z)\} \quad (6)$$

But, surface impedance for normal incidence is related to the electric and magnetic fields as follows, using Ohms law [6,7]:

$$Z_s = \frac{E_y(z)}{H_x(z)} \Big|_{z=0^+} \quad (7)$$

It can be easily seen that determining surface impedance is not so straight forward in the non-local response regime, since:

$$\{\sigma(z)*E_y(z)\}|_{z=0} \equiv \gamma(0)H_x(0) \quad (8)$$

Now, taking the Fourier transform of both sides of (6) gives the non-local constitutive equation in reciprocal q -space [6]:

$$J_c(q)|_{p=1} = \sigma(q)E(q) \quad (9)$$

Surface impedance, by its very nature, must be expressed within real space and so (9) is of no direct use. In the local response regime, however, the generalised conductivity behaves like a δ -function, i.e. $\sigma(z-z') \rightarrow \sigma\delta(z-z')$.

$$\therefore \int_{-\infty}^{+\infty} \delta(z-z')E_y(z').dz' = E_y(z) \quad \text{and} \quad J_{cy}(z)|_{p=1} \rightarrow \sigma E_y(z) \quad (10)$$

Therefore, in the local response regime, which represents the long-wavelength limit $q \rightarrow 0$ in q -space, and ignoring displacement current, the simple expression for surface impedance can be easily determined using (5), (7) and (10):

$$Z_s = \frac{\gamma(0)}{\sigma} = \sqrt{\frac{j\omega\mu_o\mu_r}{\sigma}} \quad \text{where} \quad \sigma \neq f(q) \quad (11)$$

Evidently, (11) could only have been derived here if the intrinsic bulk conductivity is in the local response regime. Since work by Pan and Zhang both adopt (11) with intrinsic bulk

conductivity having spatial dispersion, i.e. in a non-local response regime, this is direct proof of a fundamental contradiction.

3.2 Second contradiction

Angular frequency ω and modified wave number q are assumed to be real variables by Zhang [8], but this must be a complex variable when modeling normal metals at room temperatures and at frequencies below the plasma frequency, as previously stated by Lucyszyn [4]. In general, either the angular frequency ω or the modified wave number q must be a complex term, in order to account for an exponentially decaying wave as it propagates into the metal. The dispersion relations for electromagnetic waves in an isotropic homogeneous medium has either q being complex and $q=f(\omega)$, where ω is real, or ω being complex and $\omega=f(q)$, where q is real [6]. Only if there is no dissipation of electromagnetic energy can both q and ω be real [6]. Therefore, since both are treated as real variables, this contradicts the principle of conservation of energy for wave propagation within a normal metal; once again challenging the validity of the Pan and Zhang model.

4 Conclusions

It would be inappropriate to use the model given by Pan and Zhang until they can satisfactorily address the following issues: a) produce a clear and detailed derivation of (3), using (1) and (2); b) define without any ambiguity their variable q ; c) demonstrate their previous calculations of surface impedance, while showing exactly how their values of q were determined; d) explain how they managed to overcome both contradictions outlined previously; e) explain how the use of equations derived for longitudinal wave propagation can be used to create expressions for terms based on transverse wave propagation; f) explain why their model does not converge onto the classical relaxation-effect model at such low terahertz frequencies; g) give a reasonable physical insight to justify the need for a new model.

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