Plasma oscillations and terahertz instability in field-effect transistors with Corbino geometry

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Propagating between the contacts of a field-effect transistor (FET), plasma waves in its channel can become unstable and lead to generation of terahertz radiation. While previous studies of this instability concentrated on rectangular FETs, alternative geometries present fresh opportunities. We studied theoretically plasma oscillations in a gated FET with Corbino geometry where, in contrast with the rectangular FET, the oscillations become unstable at symmetric boundary conditions. Moreover, their lowest eigenfrequency is almost twice as high as that in the rectangular FET at comparable instability increments. These advantages make the Corbino FET promising for practical realizations of terahertz oscillators. © *2010 American Institute of Physics*. [doi:10.1063/1.3532850]

Due to their potential for terahertz sources and detectors, plasma waves (also known as plasmons and plasmon-polaritons) in two-dimensional electron structures have attracted increased attention in recent years. A mechanism for terahertz generation was suggested by Dyakonov and Shur[1] who noticed that plasma waves in the channel of a field-effect transistor can become unstable when propagating between the transistor's contacts. Various aspects of the Dyakonov–Shur instability have been considered subsequently: effects of collisions and diffusion,[2–4] nonuniform channels,[5,6] etc. For a recent review, see Ref. 7 and for recent experiments, see Ref. 8.

The Dyakonov–Shur instability can occur only in channels of finite length. The channel boundaries—the source and the drain—are, therefore, central in determining the conditions for the instability. In their pioneering work, Dyakonov and Shur[1] demonstrated the instability for two asymmetric boundary conditions, one requiring zero ac potential at the source and the other, zero ac conduction current at the drain. Although other boundary conditions were subsequently studied,[3,5] their asymmetry has been considered essential for the instability to occur.[7]

While being impeccable mathematically, asymmetric boundary conditions do not easily lend themselves to practical realizations. They might need either different source and drain contacts or external circuits, both complicating experimental design. Knap, Lusakowski and co-workers[9,10] also suggested that such boundary conditions can be realized by driving the transistor in saturation, but they admitted that the original theory of Dyakonov and Shur may not work in this regime. Moreover, the plasma waves in the saturated transistor can be suppressed by hot-carrier effects.[11]

Realizing the Dyakonov–Shur instability under symmetric boundary conditions could, therefore, simplify the experimental design and lead to practical terahertz sources. Whereas previous studies of the instability concentrated on rectangular field-effect transistors (FETs), we abandon the rectangular symmetry and consider instead the cylindrical geometry shown schematically in Fig. 1. In this geometry, known as a Corbino FET,[12] the source and the drain are two concentric electrodes. The two-dimensional channel occupies the space between the source and the drain, and there is a gate above it. The geometric asymmetry between the source and the drain will allow the instability to occur at symmetric boundary conditions. We concentrate on the boundary conditions corresponding to highly conducting contacts, which are important both for theory and experiment, but we discuss alternative boundary conditions as well.

The Dyakonov–Shur instability belongs to a large family of instabilities in confined plasmas. These have been known since 1944 when Pierce[13] studied electron beams drifting between two grid electrodes kept at equal potentials. The instabilities of Pierce and Dyakonov and Shur are analogous in that they both occur when an otherwise stable plasma is confined between two boundaries. Interestingly, in Pierce's instability, both space-charge waves propagate in the same di-

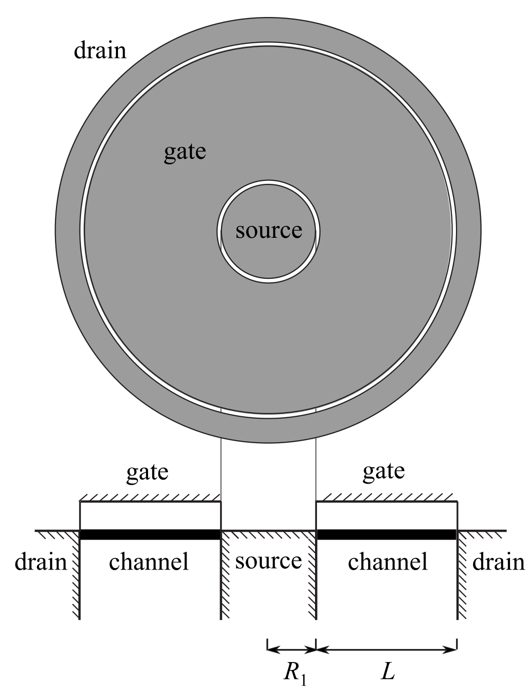

FIG. 1. The Corbino FET (top- and side-view) has concentric source and drain and a top gate.

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rection, and it was not explained as amplification of a reflected wave. Instead, an interpretation was given based on feedback between the electrodes controlled by external circuits.[14,15] Later work by Read[16] on transit time oscillators belongs to the same category. Devices based on Read's proposal have been commercially available for quite some time.

This letter presents the derivation and solution of the equations governing the plasma oscillations in the Corbino FET. It shows that the oscillations become unstable for the boundary conditions of zero ac potential both at the source and the drain, and it further discusses the eigenfrequency and the instability increment for the lowest plasma eigenmode.

To describe the plasma oscillations, we use three equations: the equation of motion, the continuity equation, and the gradual-channel equation. The equation of motion has the form

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{e}{m^*}\nabla\varphi, \tag{1}$$

where $\mathbf{v}$ is the electron velocity, e is the electron charge, m^* is the electron effective mass, and φ is the potential. Here, we used the quasistatic approximation by presenting the electric field $\mathbf{E}$ as $\mathbf{E}=-\nabla\varphi$. Also, we ignored in Eq. (1) electron collisions and diffusion.

The continuity equation has the form

$$e\frac{\partial n}{\partial t} + \nabla\cdot\mathbf{J} = 0, \tag{2}$$

where $\mathbf{J}=en\mathbf{v}$ is the current density, and n is the electron density.

The gradual-channel equation has the form[17]

$$n = \frac{C\varphi}{e}, \tag{3}$$

where C is the capacitance per unit of the gate area. This equation requires a thin dielectric layer between the channel and the gate (see Fig. 1). For plasma oscillations in a FET, this equation was used by Dyakonov and Shur[1] in their original publication, and it has been recently verified by Millithaler *et al.*[18] using Monte-Carlo calculations.

As is usually done for the analysis of instabilities, we present the potential, velocity, and density as a sum of a large dc part and a small ac part: $(\varphi,\mathbf{v},n)=(\varphi_0,\mathbf{v}_0,n_0)+(\tilde{\varphi},\tilde{\mathbf{v}},\tilde{n})\exp(j\omega t)$, $|(\tilde{\varphi},\tilde{\mathbf{v}},\tilde{n})| \ll |(\varphi_0,\mathbf{v}_0,n_0)|$; here ω is the angular frequency and j is the imaginary unit. With this assumption, we can linearize Eqs. (1)–(3) by ignoring products of small quantities; for example, for the ac density, we write $\tilde{\mathbf{J}}=en_0\tilde{\mathbf{v}}+e\tilde{n}\mathbf{v}_0$. Having done that, we can solve the equations for the ac quantities subject to boundary conditions and thus determine the spectrum of eigenfrequencies. The values of the eigenfrequencies will be, in general, complex, so that $\omega=\mathrm{Re}\,\omega+j\,\mathrm{Im}\,\omega$. Unstable oscillations are indicated by $\mathrm{Im}\,\omega<0$.

To analyze the equations for the ac quantities, we need to know their dc counterparts. According to the gradual-channel equation, Eq. (3), constant dc potential, $\varphi_0=$const, implies constant dc electron density, $n_0=$const. As follows from the continuity equation, the dc (drift) velocity should be of the form

$$\mathbf{v}_0 = \frac{\alpha}{r}\mathbf{e}_r, \tag{4}$$

where r is the radial coordinate, $\mathbf{e}_r$ is the radial unit vector, and α is a constant such that $\alpha=R_1 v_0|_{r=R_1}$. This behavior of the velocity in the Corbino FET is in contrast with that in the rectangular FET, where both the drift velocity and the density are constant along the channel.

To further simplify the analysis, we will concentrate only on the lowest-order mode, which is most likely to be of practical significance. For this mode, there is no angular dependence of any quantity, so that $\partial/\partial\theta=0$. Consequently, we consider only the radial component of the ac velocity.

With the help of the above assumptions and of Eq. (3), we can rewrite the equation of motion, Eq. (1), and the continuity equation, Eq. (2), for the ac velocity $\tilde{v}$ and potential $\tilde{\varphi}$ in the form

$$j\omega\tilde{v} - \frac{\alpha\tilde{v}}{r^2} + \frac{\alpha}{r}\frac{d\tilde{v}}{dr} = -\frac{e}{m^*}\frac{d\tilde{\varphi}}{dr},$$

$$j\omega\tilde{\varphi} + \varphi_0\frac{d\tilde{v}}{dr} + \frac{\varphi_0\tilde{v}}{r} + \frac{\alpha}{r}\frac{d\tilde{\varphi}}{dr} = 0. \tag{5}$$

These two equations require two boundary conditions, which we take in the form $\tilde{\varphi}|_{R_1}=\tilde{\varphi}|_{R_2=R_1+L}=0$.

Introducing a dimensionless coordinate $x=r/L$, a dimensionless potential $\Phi=\tilde{\varphi}/\varphi_0$, a dimensionless velocity $\psi=\tilde{v}/v_0$, and defining the plasma-wave velocity as[1] $s^2=e\varphi_0/m^*$, we recast the above equations in the form

$$\left(j\kappa - \frac{\beta}{x^2}\right)\psi + \frac{\beta}{x}\frac{d\psi}{dx} = -\frac{d\Phi}{dx},$$

$$j\kappa\Phi + \frac{\beta}{x}\frac{d\Phi}{dx} + \frac{d\psi}{dx} + \frac{\psi}{x} = 0, \tag{6}$$

where $\kappa=\omega L/s$ and $\beta=\alpha/(Ls)$. The boundary conditions can then be written as $\Phi(x_1)=\Phi(x_2)=0$, where $x_{1,2}=R_{1,2}/L$.

We proceed with the analysis of Eq. (6). In the absence of electron drift $\beta=0$, the equation for the potential Φ reduces to Bessel's equation. With the boundary conditions chosen, the equation for the lowest eigenfrequency is then $J_0(\kappa x_1)/Y_0(\kappa x_1)=J_0(\kappa x_2)/Y_0(\kappa x_2)$, where J_0 and Y_0 are zero order Bessel functions of the first and second kinds, respectively. This equation can only be solved numerically. Consequently, we also chose to solve numerically the more general boundary value problem (6) in the presence of drift.

The real and imaginary parts of the eigenfrequency depend on two parameters: the electron drift velocity and the ratio between the sizes of the channel and the source. Figure 2 shows the (a) real and the (b) imaginary parts of the eigenfrequency, $\kappa=\omega L/s$, depending on the drift velocity at the source in the Corbino FET for three values of the channel length. The dashed lines are for $L=R_1$, the dashed-dotted lines are for $L=5R_1$, and the solid black lines are for $L=20R_1$. Also shown by solid gray lines are the real and imaginary parts of the lowest eigenfrequency in the classical rectangular FET with the asymmetric boundary conditions of Ref. 1.

For all three values of L, $\mathrm{Im}\,\kappa<0$, which means the plasma oscillations are unstable; see Fig. 2(b). At low drift velocities, which are of most practical significance, the insta-

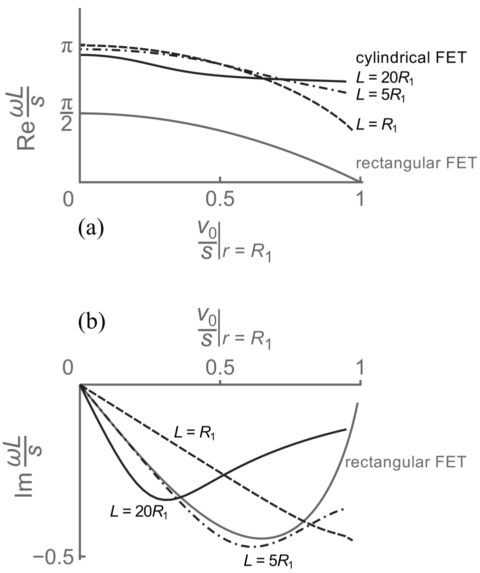

FIG. 2. (a) Real and (b) imaginary parts of the lowest eigenfrequency of plasma oscillations in the Corbino FET depending on the drift velocity at the source. The negative imaginary parts indicate unstable oscillations, which can be used for terahertz generation.

bility increment is proportional to the drift velocity, and it is larger for longer channels. For $L=R_1$, the instability increment at small drift velocities is smaller than that in the rectangular FET. For $L=5R_1$, both increments are almost the same, and for $L=20R_1$, the increment in the Corbino FET exceeds that in the rectangular FET. As our calculations show, if electrons drift in the opposite direction (in other words, if the source and the drain are interchanged), the imaginary part of the eigenfrequency is positive, and the plasma oscillations become damped instead of being unstable.

The real parts of the dimensionless eigenfrequency Re κ are close to π at low drift velocities for all three values of the channel length; see Fig. 2(a). As such, the eigenfrequencies are almost twice as large than the corresponding value in the rectangular FET, shown by gray line in Fig. 2(a). For example, for the typical plasma-wave velocity[1,3,7,9] of $s=10^8$ cm/s, the frequency of 1 THz in the Corbino FET corresponds to the channel length of approximately 500 nm.

The instability can occur also at other symmetric boundary conditions. In particular, we have studied the boundary conditions when the ac density and potential are related to each other by a pure reactance X. We found instability in a wide range of X, both positive and negative. As these results suggest, relying on geometrical source-drain asymmetry rather than on asymmetry in boundary conditions might be of general significance.

Thus, plasma oscillations in the Corbino FET can become unstable in the presence of electron drift, and this instability can be used for generating terahertz radiation. Compared with the rectangular FET, the oscillations in the Corbino FET become unstable for symmetrical boundary conditions and have higher eigenfrequencies at comparable or higher increment values. These properties could make the Corbino geometry advantageous for realization of practical terahertz oscillators. To increase the instability increment and, therefore, the output power, the size of the source should be made small comparable with the channel length. For terahertz applications, it implies nanometric sources. Practical realization of such FETs poses some obvious difficulties, particularly in lithography.

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