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From the W Boson Mass Anomaly to Little Higgs Model

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Abstract

With the recent announcement of a deviation in measuring the W boson mass M_W from the Standard Model (SM) expected value carried out by CDF collaboration, many theories concentrating on Beyond Standard Model (BSM) mechanism are now available to testify their predictions. In this article, we introduce Little Higgs Model , which may imply and forecast new physics on multi-TeV scale, and perform fit with the recent results on W boson mass anomaly to test the compatibility. The results can give guidance to prospective phenomenology research and the construction of new colliders for High Energy Physics (HEP) discoveries.

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Contents

Chapter 1

Introduction: Standard Model Revisited

In April 2022, the CDF collaboration finished the digging of data from the CDF II experiment carried out in the 2000's and announced an anomaly about the W boson mass M_W [1] as

$$M_W = 80.433 \pm 0.0064_{statistical} \pm 0.0069_{system}(GeV^2) \tag{1.1}$$

This result, on the one hand, is the most precise measurement based on enormous data collected during the Run II phase. On the other hand, however, the SM prediction, in correspondence with other experiments, reads [2]

$$M_W = 80.357 \pm 0.004_{input} \pm 0.004_{theory}(GeV^2) \tag{1.2}$$

in which the 'input' variance contains perturbation expansion errors arising from other essential parameters including the Z and Higgs boson masses, the top-quark yukawa coupling, the electromagnetic coupling and the muon lifetime. As Eq (1.1) shows, the measured M_W has a deviation as large as 7σ, indicating a discrepancy with SM and other experiments, with a more comprehensive and contrasting figure given in Fig 1.1. Although the further verification of data on Large Hardon Collider (LHC) is indispensable to testify the result, the self-consistency of the anomaly encourages us to assume the correctness of such M_W measurement, generating a new era for Beyond Standard Model (BSM) physics search.

Why is the precise measurement of M_W so important? The answer lies in the overall overlook of the ultimate objective of the high energy and theoretical physics, where people try to find a theory or several theories suitable to explain phenomena taking place in all energy scale. Among them, Standard Model(SM) is just treated as a low energy effective theory, but it is also the most successful and most thoroughly explained theory. Therefore, as an essential parameter of SM, the precise measurement of M_W should provide powerful constraints for higher energy theories, serving as a stepping stone. In this dissertation, the core will be the introduction to a powerful TeV scale theory: the little higgs, where the new M_W anomaly will be plugged in to give directional suggestions for phenomenology research based on selected little higgs model.

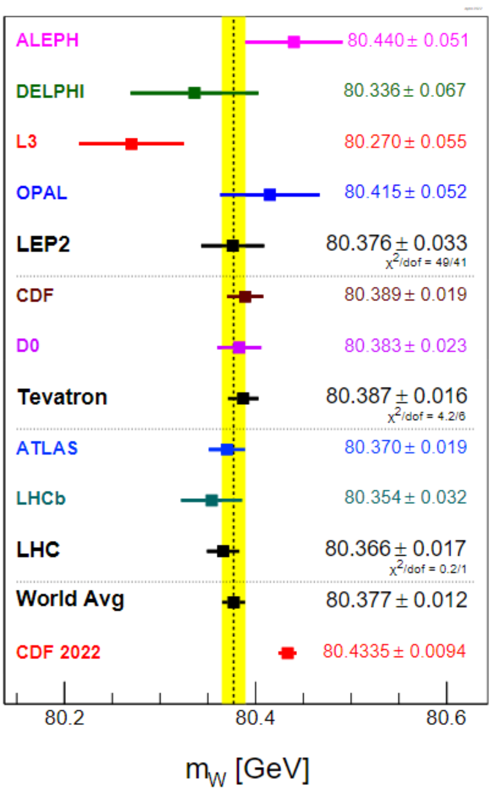


Figure 1.1: The current M_W spectrum measured by previous HEP experiments[3]

The paper will start from reviewing the W boson originality from SM and the well-known Higgs Mechanism (Chapter 1), in comparison with the following proposed Little Higgs Model (Chapter 2). Then after the introduction on the postulates about the Little Higgs Model, the M_W anomaly will be fitted in (Chapter 3) to give further predictions on other prospects of current particle physics (Chapter 4). A conclusion and outlook will be present at the end of the paper (Chapter 5).

1.1 Higgs Mechanism and electroweak symmetry breaking

We will begin with a retrospection on Spontaneous Symmetry Breaking (SSB) and the famous Higgs Mechanism, which serves as an indispensable bridge connecting the electromagnetism and weak interaction, known as electroweak theory or Glashow-Weinberg-Salam Theory.

The SSB will be introduced via a simple model – considering a complex scalar field ϕ with a global abelian $U(1)$ symmetry. The Lagrangian reads under potential $V(|\phi^*\phi|)$ as

$$L(\phi) = \partial_\mu\phi^*\partial^\mu\phi - V(|\phi^*\phi|) = \partial_\mu\phi^*\partial^\mu\phi + \mu^2|\phi^*\phi| - \frac{\lambda}{2}|\phi^*\phi|^2 \tag{1.3}$$

where the μ has the dimension of and will be interpreted as mass, and λ is a coefficient. The potential V clearly has a non-zero Vacuum Expectation Value (VEV), meaning the minimum happens at $\phi_0 \neq 0$

$$\frac{\partial V}{\partial \phi^*} = 0 \Rightarrow |\phi_0^*\phi_0| = \frac{\mu^2}{\lambda} = \frac{v^2}{2} \tag{1.4}$$

Choosing the $\phi_0 = v/\sqrt{2}$, the corresponding scalar field near ϕ_0 is

$$\phi = \frac{1}{\sqrt{2}}(v + \varphi + \mathrm{i}\chi) \tag{1.5}$$

where the real part φ and imaginary part χ from the complex scalar are separated. After inserting Eq (1.5), the potential now becomes

$$V = -\mu^2|\phi^*\phi| + \frac{\lambda}{2}|\phi^*\phi|^2 = -\frac{\mu^4}{2\lambda} + \mu^2\varphi^2 + (\textit{higher order}) \tag{1.6}$$

Eq (1.6) indicates that: firstly, the $U(1)$ symmetry for ϕ has been broken after a specific VEV is chosen, therefore we can say the symmetry is spontaneously broken; moreover, after the symmetry is breaking, the imaginary scalar part generates no mass at all; therefore, the 1 broken degree of freedom transforms into a massless boson, conserving the overall degree of freedom. Such generated massless scalar is called Goldstone boson, or **Nambu-Goldstone Boson (NGB)**. Such pattern is typical in more generalized symmetry breaking process G/H, where G and H are respectively the symmetry group before and after, and 'G/H' is just the coset of those missing NGBs appearing from the SSB process. This notation will become quite useful in Chapter 2 discussing the specific little higgs model, with a more complicated non-abelian symmetry breaking.

The Higgs Mechanism is just a process of SSB, but along with the broken symmetry group being the gauge group – considering a composite complex scalar field

$\phi_i = (\phi_1, \ldots, \phi_n)^T$, with local symmetry G. The Lagrangian reads

$$L = c\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + (D_\mu\phi_i)^\dagger(D^\mu\phi_i) - V(\phi_i) \tag{1.7}$$

where $F_{\mu\nu}$ is the field strength with gauge field A obeying G symmetry, c is a normalizing constant, and $D = \partial - \mathrm{i}gA$ is the covariant derivative.
Now a SSB condition is applied to the system, with remnant unbroken group H. The perturbative $\phi = \phi_0 + \varphi$ near the VEV $\phi_0 \neq 0$ will make the covariant derivative go as

$$D_\mu\phi_i = \partial_\mu\varphi_i - \mathrm{i}gA^a_\mu T^a(\phi_0 + \varphi_i) \tag{1.8}$$

Using the fact that the VEV can be split into the G generator T^a as $\phi^a_0 = T^a\phi_0$, where they are orthogonal to each other. Then the corresponding Lagrangian term now looks like

$$\begin{aligned}(D_\mu\phi_i)^\dagger(D^\mu\phi_i) =&(\partial_\mu\varphi_i)^\dagger(\partial^\mu\varphi_i) - \mathrm{i}g(\partial\varphi_i)^\dagger A^{\mu a}\phi^a_0 + \mathrm{i}gA^{a*}_\mu\phi^{a\dagger}_0(\partial^\mu\varphi_i)\\ &+ g^2\lambda_a A^{a*}_\mu A^{\mu a} + (\mathit{higher\ order})\end{aligned} \tag{1.9}$$

in which we use the orthogonality of $\phi^a_0\phi^b_0 = \lambda_a\delta^{ab}$. To avoid mixing in φ and A in the Lagrangian terms, we can cancel the mixing by introducing the unitary gauge as

$$\varphi^\dagger\phi^a_0 = 0 \tag{1.10}$$

Eq (1.10) implies that for every non-zero ϕ_0 corresponding to a NGB mode, the resulting perturbative field φ is eliminated from the Lagrangian. The degree of freedom is no longer inside the scalar field and transported into the massive gauge field as shown in Eq (1.9). Therefore, we can see that the Higgs Mechanism transmits the generated goldstone boson from SSB to the vector fields and gives them mass.

Serving as the foundation of Standard Model, the electroweak theory provides a good instance as the implication of Higgs Mechanism into these two interactions. The symmetry breaking pattern is now $SU(2) \times U(1)_{hyper}/U(1)_{EM}$, indicating a $SU(2) \times U(1) \to U(1)$ structure, where the $SU(2)$ is the isospin group in weak interaction and $U(1)_{hypercharge}$ is the abelian group conserving the hypercharge, and $U(1)_{EM}$ is the usual electromagnetism interaction.
In such circumstance with $SU(2) \times U(1)$, the covariant derivative term in Eq (1.8) is now

$$D_\mu\phi = \partial_\mu\phi - \mathrm{i}\left(gW^a_\mu\tau^a + \frac{1}{2}g'B_\mu\right)\phi \tag{1.11}$$

in which the ϕ is a two component complex scalar field, named as Higgs Field, with a VEV as

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{1.12}$$

and the W^a, B are respectively the gauged boson field $SU(2)$ and $U(1)_{hyper}$, the τ^a are the generators of $SU(2)$ group, $a = 1, 2, 3$ separately ,which gives

$$\tau^a = \frac{1}{2}\sigma^a \qquad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1.13}$$

The σ^a matrix is the usual Pauli matrix. The gauge coupling for these two fields are g and g' respectively, where it should be noticed that a conventional $1/2$ on hypercharge field is multiplied.
Using Eq (1.11) and Eq (1.12), the Lagrangian term in Eq (1.9) can be expanded into

$$\Delta L = \frac{1}{2} \begin{pmatrix} 0 & v \end{pmatrix} (gW^a_\mu \tau^a + \frac{1}{2} g' B_\mu \mathbb{1})(gW^{\mu b}\tau^b + \frac{1}{2} g' B^\mu \mathbb{1}) \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{1.14}$$

by implementing the Pauli matrix Eq (1.13), the expression after simplification is derived as

$$\Delta L = \frac{1}{2}\frac{v^2}{4}[g^2 W^1_\mu W^{\mu 1} + g^2 W^2_\mu W^{\mu 2} + (gW^3_\mu - g'B_\mu)(gW^{\mu 3} - g'B^\mu)] \tag{1.15}$$

where the identity $\sigma^a \sigma^b = \delta^{ab} + \mathrm{i} f^{abc}\sigma^c$ can help in simplifying the equation.
Eq (1.15), however, is still not in the mass eigenstates, therefore requiring further derivations. Rewrite it in the matrix form with basis $A_\mu = (W^1_\mu, W^2_\mu, W^3_\mu, B_\mu)$yields

$$\Delta L = \frac{1}{2}\frac{v^2}{4} \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix} \tag{1.16}$$

The calculation gives the mass eigenstates with eigenvector, the broken massive vector bosons as the following:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp \mathrm{i} W^2_\mu) \qquad \textit{with mass } m_W = \frac{gv}{2} \tag{1.17}$$

$$Z^0_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(gW^3_\mu - g'B_\mu) \qquad \textit{with mass } m_Z = \frac{\sqrt{g^2 + g'^2}v}{2} \tag{1.18}$$

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g'W^3_\mu + gB_\mu) \qquad \textit{with mass } m_\gamma = 0 \tag{1.19}$$

where we can see that the $SU(2) \times U(1)/U(1)$ generates 3 NGBs from the broken phase of the scalar field ϕ, and immediately absorbed by the massive $W_\mu^\pm$, Z_μ^0, while the fourth gauge field A_μ remains massless, which is just the classical electromagnetic field with propagator massless photon. And the higgs field ϕ abridges the symmetry between the two interactions.

1.2 M_W predicted by SM

The Eq (1.17) provides a first-place prediction on the W Boson Mass. However, further improvement and refinement is available even within the frame of Standard Model.

To begin with, considering the $SU(2) \times U(1)$ gauge symmetry for a general field, with $U(1)$ charge$Y$ and under a $SU(2)$ general representation T^a , $a = 1, 2, 3$

$$D_\mu = \partial_\mu - \mathrm{i}\left(gW_\mu^a T^a + g'YB_\mu\right) \tag{1.20}$$

Compared with Eq (1.11), we can see that the Higgs Boson is just the specialization with charge $Y = 1/2$.

It is useful to change Eq (1.20) into mass eigenbasis, which gives

$$D_\mu = \partial_\mu - \mathrm{i}\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - \mathrm{i}\frac{1}{\sqrt{g^2+g'^2}} Z_\mu^0 (g^2 T^3 - g' Y) - \mathrm{i}\frac{gg'}{\sqrt{g^2+g'^2}} A_\mu (T^3 + Y) \tag{1.21}$$

where the $T^\pm = T^1 \pm \mathrm{i}T^2$, and we can choose for simplicity and normalization $T^\pm = (\sigma^1 \pm \mathrm{i}\sigma^2)/2$. Moreover, since we recognize the A as the EM field, the unit charge e and electric charge number Q can be identified as

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} \tag{1.22}$$

$$Q = T^3 + Y \tag{1.23}$$

For measurements on the transformation of weak interaction eigenstates to broken mass eigenstates, we introduce a parameter called **Weak Mixing Angle** θ_W

$$\cos^2\theta_W = \frac{g^2}{g^2+g'^2} \qquad \sin^2\theta_W = 1 - \cos^2\theta_W = \frac{g'^2}{g^2+g'^2} \tag{1.24}$$

On the other hand, from Eq (1.17) and Eq (1.18) we can also recognize

$$\cos^2\theta_W = \frac{m_W^2}{m_Z^2} \tag{1.25}$$

Finally, by using Eq (1.22), Eq (1.23) and Eq (1.24), we can write Eq (1.20) into a more intuitive form

$$D_\mu = \partial_\mu - \mathrm{i}\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - \mathrm{i}\frac{g}{\cos\theta_W} Z_\mu^0 (T^3 - \sin^2\theta_W Q) - \mathrm{i}eA_\mu Q \quad (1.26)$$

where

$$g = \frac{e}{\sin\theta_W} \quad (1.27)$$

is just an alternative expression of g by taking Eq (1.24) into Eq (1.22).
Then we can consider the Standard Model fermionic coupling under subgroup $SU(2)\times U(1)$ with covariant derivative Eq (1.26). The kinetic term of the full SM Lagrangian is

$$L_{kin} = \bar{\ell}_L \mathrm{i}\not{D}\ell_L + \bar{e}_R \mathrm{i}\not{D} e_R + \bar{Q}_L \mathrm{i}\not{D} Q_L + \bar{u}_R \mathrm{i}\not{D} u_R + \bar{d}_R \mathrm{i}\not{D} d_R \quad (1.28)$$

where the slashed covariant derivative $\not{D} = \gamma^\mu D_\mu$, the typical fermion derivative abbreviation with γ the gamma matrix. And $\ell_L = (\nu_L \quad e_L)^T, e_R, Q_L = (u_L \quad d_L)^T, u_R, d_R$ are the usual SM fermions with chirality.
After inserting Eq (1.26), the lagrangian terms become

$$\begin{aligned} L_{kin} =& \bar{\ell}_L \mathrm{i}\not{\partial}\ell_L + \bar{e}_R \mathrm{i}\not{\partial} e_R + \bar{Q}_L \mathrm{i}\not{\partial} Q_L + \bar{u}_R \mathrm{i}\not{\partial} u_R + \bar{d}_R \mathrm{i}\not{\partial} d_R \\ &+ g(W_\mu^+ J_W^{\mu+} + W_\mu^- J_W^{\mu-} + Z_\mu^0 J_Z^\mu) + eA_\mu J_{EM}^\mu \end{aligned} \quad (1.29)$$

The corresponding electroweak currents are

$$\begin{aligned} J_W^{\mu+} &= \frac{1}{\sqrt{2}}(\bar{\nu}_L\gamma^\mu e_L + \bar{u}_L\gamma^\mu d_L) \\ J_W^{\mu-} &= \frac{1}{\sqrt{2}}(\bar{e}_L\gamma^\mu \nu_L + \bar{d}_L\gamma^\mu u_L) \\ J_Z^\mu &= \frac{1}{\cos\theta_W}[\frac{1}{2}\bar{\nu}_L\gamma^\mu\nu_L + \bar{e}_L\gamma^\mu(-\frac{1}{2} + \sin^2\theta_W)e_L + \bar{e}_R\gamma^\mu \sin^2\theta_W e_R \\ &+ \bar{u}_L\gamma^\mu(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)u_L - \bar{u}_R\gamma^\mu\frac{2}{3}\sin^2\theta_W u_R \\ &+ \bar{d}_L\gamma^\mu(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W)d_L + \bar{d}_L\gamma^\mu\frac{1}{3}\sin^2\theta_W d_L] \\ J_{EM}^\mu &= \bar{e}\gamma^\mu(-1)e + \bar{u}\gamma^\mu(\frac{2}{3})u + \bar{d}\gamma^\mu(-\frac{1}{3})d \end{aligned} \quad (1.30)$$

Such currents may have implications in particle scattering process in weak interaction with virtual W boson as the propagator. For example, considering the tree-level Feynmann diagram shown in Fig 1.2. The propagator has form like

$$\frac{-\mathrm{i}g^{\mu\nu}}{p^2 - m_W^2} \quad (1.31)$$

If we assume the reaction happens at low energy, where $p^2 << m_W$), then the

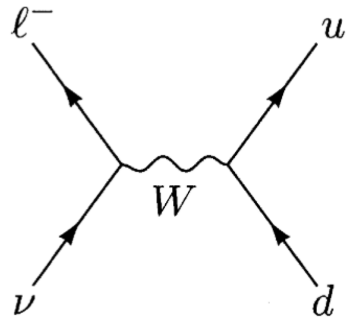


Figure 1.2: A typical process with W as the propagator

scattering process can be expressed via the effective Lagrangian

$$\Delta L_W \approx \frac{g^2}{2m_W^2}(\bar{\nu}_L\gamma^\mu e_L + \bar{u}_L\gamma^\mu d_L)(\bar{e}_L\gamma^\mu \nu_L + \bar{d}_L\gamma^\mu u_L) = \frac{g^2}{m_W^2} J_W^{\mu+} J_W^{\mu-} \tag{1.32}$$

through which we can define the **Fermi Constant** G_F

$$G_F = \frac{g^2}{4\sqrt{2}m_W^2} \tag{1.33}$$

However, on the other hand, we have Eq (1.25) and Eq (1.27). Employing the **fine structure constant** α with $\alpha = e^2/4\pi$ (where we adhere to the natural unit $c_0 = \hbar = 1$), we can get an alternative expression of M_W

$$m_W^2\left(1 - \frac{m_W^2}{m_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F} \tag{1.34}$$

Eq (1.34) now provides a more accurate expectation value on W boson mass from tree-level scattering. Though the form is similar to Eq (1.25), more of the included observable variables ensures a more accurate value on the mass. More fine process can be made if loop-order propagation has been included, resulting in

$$m_W^2\left(1 - \frac{m_W^2}{m_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1 + \Delta r) \tag{1.35}$$

Precise calculation of one-loop and higher order terms can be found in Ref [4] and Ref [5], which is beyond the scope of the focus of the dissertation.

1.3 Onto the road of little higgs

In the previous sections we briefly reintroduced the electroweak theory, while presenting procedures to get the SM predicted W boson mass term as Eq (1.2). Nev-

ertheless, as mentioned in the beginning, the result is in conflict with the latest published experiment measurements. This brings us back to the problem: *Is Standard Model flawless in describing the particle family and the world?*
The answer, definitely, is no. Despite the finding of CDF collaboration, with the development and application of more advanced and accurate detector and accelerator, the SM has been observed to provide unsatisfactory or inaccurate explanations to some observed phenomena, including the Neutrino Mass Hierarchy, CP violation, etc. Therefore, to bring explanation to these observations, new theories must be included to modify the SM.
On the other hand, the SM, a gauge theory only governing low energy – electroweak scale $\sim m_W$, has been avoiding to demonstrate on more generic problems, including

- How would the physics be at higher energy level Λ, such as gravitation at planck scale?
- (consequently)What will the physics be like at energy between the current electroweak scale m_W to Λ? Will there be new mechanism or new physics to be discovered?
- Why Higgs itself can not be a NGB instead of the current massive one?

Knowing that the SM is more or less a low-energy effective field theory, such problem is called *hierarchy problem*, focusing mainly on the scenario of physical description in much higher energy scale. Throughout the years, theoretical physicists have proposed many extended theories working in higher energy world, including supersymmetry (SUSY), the grand unified theory(GUT) etc. Nevertheless, the stringent constraint is that the theory must converge to and be compatible with the current electroweak-scale experiment measurements. Therefore, there are theories instead focusing on the energy just above the m_W scale, namely in TeV scale, to firstly throw light on the second question. A promising and motivating theory, reformulating and generalizing the electroweak theory into TeV scale, is the so-called *Little Higgs Model*, which is identified mainly with the following characteristics[6]

- Higgs bosons are NGBs, which spontaneously break at higher energy scale f, which is proposed to be in TeV scale.
- (Subsequently) At energy down to electroweak scale, the Higgs boson acquires mass and becomes pNGB (pseudo-Nambu-Goldston Boson)
- The Higgs bosons shall be light in energy between the two scales, meaning radiative divergence shall be cancelled via some mechanism

The postulates listed above smooth out the discrepancy between the two energy scale, nearly solved the third problem by suppose the higgs boson as pNGBs, where the 'little' originated. Moreover, it can give predictions at TeV scale, therefore providing more traces to a more unified theory. In the following chapter, we will introduce how the little higgs model is established, the challenges it faces and the approaches to eliminate the divergence.

Chapter 2

Little Higgs Model: Preliminaries and Set-ups

In this chapter, the Little Higgs Model shall be explored with firstly introductions on preliminaries of the mathematical description of the field in nonlinear sigma model (nlσm) and spontaneous symmetry breaking under such notation. Then the corrections from Lagrangian, fermion loops and other possible origins will be discussed in a minimal $SU(3)/SU(2)$ model as a starting point. And a final approach will be the littlest little higgs (L^2H) and some alternative models with real electroweak applications.

2.1 Higgs field in Nonlinear Sigma Model

The sigma model first is introduced to better analyse the meson in consequence of SSB[7], where the symmetry is divided into chiral symmetry product, eg. $SU(2)_L \times SU(2)_R$, and spontaneously broken to a third $SU(2)$ group, which is interpreted as the custodial symmetry in SM. Such motivation has been applied to consider the $\pi^{\pm}$, which is nearly massless therefore identified as pNGB. As mentioned in the previous section, the little higgs get the inspiration thus also is established into non-linear realization, which is the generalization of sigma model, called non-linear sigma model, especially in Section 2.3, where we will depict the product group model.
The sigma model starts with, as above, the discrimination of chiral symmetry $G = G_L \times G_R$, with transforming matrix L and R, in description of a general fermionic field

$$\psi = \psi_L + \psi_R \tag{2.1}$$

resulting in

$$\begin{aligned} \psi_L &\to L\psi_L \\ \psi_R &\to R\psi_R \end{aligned} \tag{2.2}$$

The infinitesimal transformation for ψ under G, shall be able to decompose into ψ_L under G_L and ψ_R under G_R, respectively giving

$$\begin{aligned} \delta\psi_L &= \mathrm{i}\epsilon_L^a T^a \psi_L \\ \delta\psi_R &= \mathrm{i}\epsilon_R^a T^a \psi_R \end{aligned} \tag{2.3}$$

$$\delta\psi = \mathrm{i}\epsilon^a T^a \psi = \mathrm{i}(\epsilon_0^a - \gamma^5 \epsilon_5^a) T^a \psi \tag{2.4}$$

where the T_a are the generators of G, with ϵ_L, ϵ_R the coefficients. In Eq (2.4) we define

$$\begin{aligned} \epsilon_0^a &= \frac{\epsilon_L^a + \epsilon_R^a}{2} \\ \epsilon_5^a &= -\frac{\epsilon_L^a - \epsilon_R^a}{2} \end{aligned} \tag{2.5}$$

to clarify the distinction of chirality and isospin states. When $\epsilon_5^a = 0$ the system is in pure isospin symmetry; $\epsilon_0^a = 0$ reflects the pure chiral case.

Under such chiral symmetry, the Lagrangian can be write explicitly with chiral fermionic field with a spinless **sigma field** Σ as

$$L = \mathrm{i}\bar{\psi}\partial\!\!\!/\psi - g\bar{\psi}_L \Sigma \psi_R - g\bar{\psi}_R \Sigma^\dagger \psi_L + L(\Sigma) \tag{2.6}$$

where the sigma field satisfies

$$\delta\Sigma = \mathrm{i}\epsilon_L^a T^a \Sigma - \mathrm{i}\Sigma\epsilon_R^a T^a \tag{2.7}$$

which corresponds to a non-infinitesimal transformation under the broken subgroup H in an adjoint representation

$$\Sigma \to L\Sigma R^\dagger \tag{2.8}$$

We can see from Eq (2.6) that the definition of Σ originated from the Higgs field sector, compared with the yukawa coupling terms in standard SM lagrangian. Under the limit $g' \to 0$, the sigma field just has the interpretation as the approximate massless meson.

In a more generalized case, under the spontaneous broken G/H, the group elements of G can be written as

$$g = e^{\mathrm{i}c^l P_l} h \qquad \textit{where } P \subset G/H, h \subset H \tag{2.9}$$

where c^l are coefficients parameterizing the coset space G/H, while P_l being the representation. One can easily see that any group element g can be represented in such a equation.

It can be demonstrated that if g is simply an element from the subgroup, the transformation of Σ is linear; but if g is from the coset, the transformation now is non-linear, corresponding to a generalized **non-linear sigma model**. A detailed introduction on it will be presented in Appendix A.

In the following sections, we will take the little higgs field as a non-linear sigma field Σ with the form like

$$\Sigma = e^{\mathrm{i}\Pi_a X_a} \Sigma_0 \tag{2.10}$$

with Σ_0 the VEV in subgroup H.

2.2 Constructing a simplest LH: $SU(3)/SU(2)$ as a example

In this section we will build an elementary little higgs model as an example to show how the mechanism is cultivated and the methods of computing the divergence. Take a $SU(3)/SU(2)$ instance, where the spontaneous broken VEV reads

$$\Sigma_0 = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \tag{2.11}$$

which is similar as Eq (1.12), the f interpreted as certain energy scale under which the spontaneous symmetry breaking is taking place. The difference is just that we are now using Eq 2.10 notation, with the Lagrangian built with sigma field

$$\Sigma(x) = \frac{1}{f} e^{\frac{2\mathrm{i}\pi^a X^a}{f}} \Sigma_0 \tag{2.12}$$

where the X^as are the broken $SU(3)$ generator, π^a the coefficient, and $a = \{1, \ldots, 5\}$ corresponds to the number of the broken degrees of freedom. They will have a form like

$$2\pi^a X^a = \begin{pmatrix} 0_{2\times 2} & H(x) \\ H(x)^\dagger & 0 \end{pmatrix} + \frac{s(x)}{2\sqrt{2}} \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -2 \end{pmatrix} \tag{2.13}$$

where the matrices are block matrices, with $H(x)$ a 2-dimensional vector, and $0_{2\times 2}$, $\mathbb{1}_{2\times 2}$ are separately 2×2 zero matrix and unit matrix. The sigma field Eq (2.12) satisfies

$$\Sigma^\dagger \Sigma = \mathbb{1} \tag{2.14}$$

After the declaration of the sigma field under such condition, we can turn to the Lagrangian to consider the divergence and determine the energy scale f.

2.2.1 Gauge coupling

Again, for the gauge coupling we will first consider the kinetic term in the lagrangian

$$L_{kin} = f^2 \partial_\mu \Sigma^\dagger \partial^\mu \Sigma \tag{2.15}$$

the constant f^2 is chosen to normalize the kinetic term, i.e. to cancel the coefficient in Eq (2.12).
Under the unbroken subgroup $SU(2)$, the derivative shall be transformed into covariant derivative with gauge field W^a_μ

$$D_\mu = \partial_\mu - \mathrm{i} g W^a_\mu Q^a \tag{2.16}$$

in which the Q^a are generators of $SU(2)$, $a = \{1, 2, 3\}$

$$Q^a = \begin{pmatrix} \frac{\sigma^a}{2} & 0 \\ 0 & 0 \end{pmatrix} \tag{2.17}$$

The σ^a is the usual pauli matrices.
Considering the gauge coupling, we can expand the sigma field to the first order

$$\Sigma = \frac{1}{f}\left[\mathbb{1} + \frac{\mathrm{i}}{f}\begin{pmatrix} \frac{s(x)}{2\sqrt{2}}\mathbb{1}_{2\times 2} & H(x) \\ H(x)^\dagger & -\frac{s(x)}{\sqrt{2}} \end{pmatrix}\right]\begin{pmatrix} \vec{0} \\ f \end{pmatrix} = \frac{1}{f}\left[\begin{pmatrix} \vec{0} \\ f \end{pmatrix} + \mathrm{i}\begin{pmatrix} H(x) \\ -\frac{s}{\sqrt{2}} \end{pmatrix}\right] \tag{2.18}$$

where we implement Eq (2.13). The 5 NGBs broken by the $SU(3)/SU(2)$ are respectively, 4 in the complex 1×2 Higgs vector, 1 in the remaining scalar s.
Inserting Eq (2.18) into the kinetic term of the lagrangian

$$\begin{aligned} L_{kin} &= f^2(\partial_\mu \Sigma^\dagger + \mathrm{i} g W^{a*}_\mu \Sigma^\dagger Q^{a\dagger})(\partial^\mu \Sigma - \mathrm{i} g W^{a\mu} Q^a \Sigma) \\ &= \left(\partial_\mu H^\dagger + \frac{\mathrm{i}}{2} g W^{a*}_\mu H^\dagger \sigma^{a\dagger}\right)\left(\partial^\mu H - \frac{\mathrm{i}}{2} g W^{a\mu} \sigma^a H\right) + L(s) \end{aligned} \tag{2.19}$$

where the terms about the parameter s are separated in the latter equation. Qualitatively, Eq (2.19) shows that the kinetic term will generate a mass term like $\mu^2 H^\dagger H$. For higher-order expansion of Eq (2.18), a quartic term $\lambda(H^\dagger H)^2$ will be included.

The mass term results in a quadratic divergence (shown in Fig 2.1) according to the Coleman-Weinberg potential by gauge boson field loop like

$$\mu^2 = c\frac{g^2}{16\pi^2}\Lambda^2 \tag{2.20}$$

where we put all the non-important terms into the coefficient c, and Λ is the high energy cutoff. A detailed instruction of how the Coleman-Weinberg potential takes

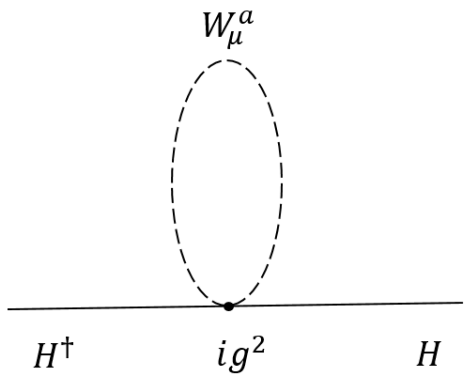


Figure 2.1: The one-loop Feynmann Diagram of the contribution to the higgs mass term

effect in the process is introduced in Appendix B. For the quartic loop correction, the term similarly looks like

$$\lambda = c' \frac{g^2}{f^2 16\pi^2} \Lambda^2 \tag{2.21}$$

Therefore, after analysis on the gauge coupling, we can get the leading quadratically divergent term in Eq (2.20) and the quartic divergent term as Eq (2.21). These are the terms we need to fix and finally get back to the SM electroweak breaking under certain constraints on the scaling factor f.

2.2.2 Fermionic yukawa coupling

Similarly, we also need to consider the yukawa interaction between the fermionic terms and the higgs coupling, which will result a diagram just like Fig 2.2, where the Q, L are the SM left-handed leptons, and the i denotes the generation of the fermion, with compensating u^c, d^c, e^c charge conjugate fermion.

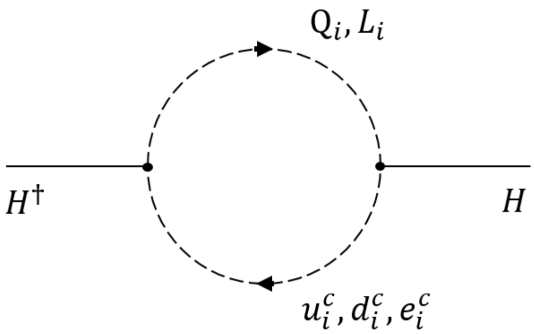


Figure 2.2: The one-loop correction of the quark and lepton yukawa coupling with Higgs field, the dashed line denoting the fermions as propagators

The corresponding loop-diagram contribution is

$$\mu^2 = c_i \frac{y_i^2 N_i^2}{16\pi^2} \Lambda^2 \tag{2.22}$$

under the high energy cutoff Λ. The y_i are the yukawa coupling for different quarks and leptons, while $N_i = 1$ for lepton and $N_i = 3$ for quark. However, the yukawa coupling constant is proportional to the mass of the fermion, and the heaviest top quark $M_t \approx 170(GeV/c^2)$ is about 5 order of magnitude larger than the lightest up quark $m_u \approx 2.16(MeV/c^2)$[3]. In fact, the $y_i \leq 0.03$ for all quarks and leptons except the top quark, whose yukawa coupling $y_t$is at order 1. Therefore, we shall consider only the top quark yukawa coupling, ignoring the side effects of all other leptons.

2.2.3 Collective symmetry breaking

From the discussion above, we can see that the gauge coupling Eq (2.20) and top quark yukawa coupling Eq (2.22) both have quadratically divergent terms. A mechanism shall be included to cancel the divergence, otherwise there will be another 'UV disaster'. The discrete concept is called **collective symmetry breaking**. We will discover how the process eliminate the mass divergence in the current $SU(3)$ example.
Consider, instead, the doubled broken symmetry $[SU(3)/SU(2)]^2$, where two sets of sigma fields like Eq (2.12) are generated

$$\begin{aligned} \Sigma_1 &= \frac{1}{f} e^{\frac{i\Pi_1}{f}} \begin{pmatrix} \vec{0} \\ f \end{pmatrix} \\ \Sigma_2 &= \frac{1}{f} e^{\frac{i\Pi_2}{f}} \begin{pmatrix} \vec{0} \\ f \end{pmatrix} \end{aligned} \tag{2.23}$$

where we use Π_i to denote the broken generator form Eq (2.13). For simplicity, we assume they have the same breaking energy scale f.
The corresponding Lagrangian kinetic term now contains two pieces

$$L_{kin} = f^2 (D_\mu \Sigma_1^\dagger D^\mu \Sigma_1 + D_\mu \Sigma_2^\dagger D^\mu \Sigma_2) \tag{2.24}$$

Then a similar calculation method as we did in Section 2.2.1 can be implemented to deal with the gauge coupling. Nevertheless, the Lagrangian now does not contain any quadratically divergent term to one loop like in Fig 2.1, since the two sigma field will cancel each other. However, the one-loop containing both of the field shall be mentioned with more concentration.

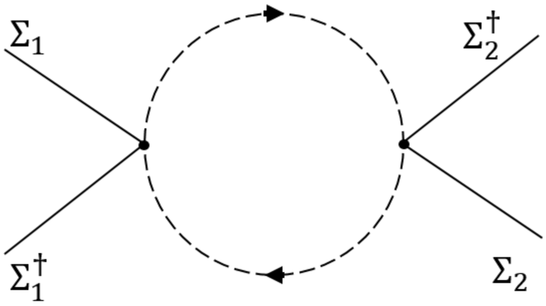


Figure 2.3: The one-loop contribution to the mixing two sigma field

As shown in Fig 2.3, the coupling between two sigma field will yield a divergent mixing term in the diagram

$$\frac{g^4}{16\pi^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)|\Sigma_1^\dagger\Sigma_2|^2 \tag{2.25}$$

which, though shown implicitly, does not involve a quadratically divergent term about the $H^\dagger H$, but with a logarithmically divergent one. The specific method can contribute to eliminating the quadratically divergence.
Consequently, given that the one-loop correction has been canceled, the gauge coupling constant of either field (since in the example they are identical) can be set to 0, resulting 10 NGBs popping up, but half of them will be absorbed into the gauge field just like ordinary higgs mechanism. The remaining 5 free ones turn out to be the exact massless NGBs, or the little higgs generated. Any other terms participate into the mass of higgs must contain the mixing of two sigma fields, nevertheless as shown in Eq (2.25), they can not generate a vicious quadratically divergent term.

For the fermionic divergence, the pattern is the same. However, a new top quark T will be included into the quark family, in order to make a triplet $\Psi = \{t, b, T\} = \{Q, T\}$ under the doubled $SU(3)$ original group, where the Q is simply the third generation of SM quark. The corresponding yukawa coupling Feynmann diagram Fig 2.4 composes of two fragments, which can be seen from the yukawa term

$$L_{yuk} = \lambda_1\Sigma_1^\dagger\Psi t_1^c + \lambda_2\Sigma_2^\dagger\Psi t_2^c \sim \lambda f\left(1-\frac{H^\dagger H}{2f^2}\right)TT^c + \lambda H^\dagger Q t^c \tag{2.26}$$

The divergence from the two diagrams can be calculated as

$$\frac{y_t^2}{16\pi^2}\Lambda^2 H^\dagger H + \frac{y_t^2 f^2}{16\pi^2}\left(1-\frac{H^\dagger H}{f^2}\right)H^\dagger H = \text{const.} \tag{2.27}$$

Therefore, it can be acquired that under collective symmetry breaking $[SU(3)/SU(2)]^2$

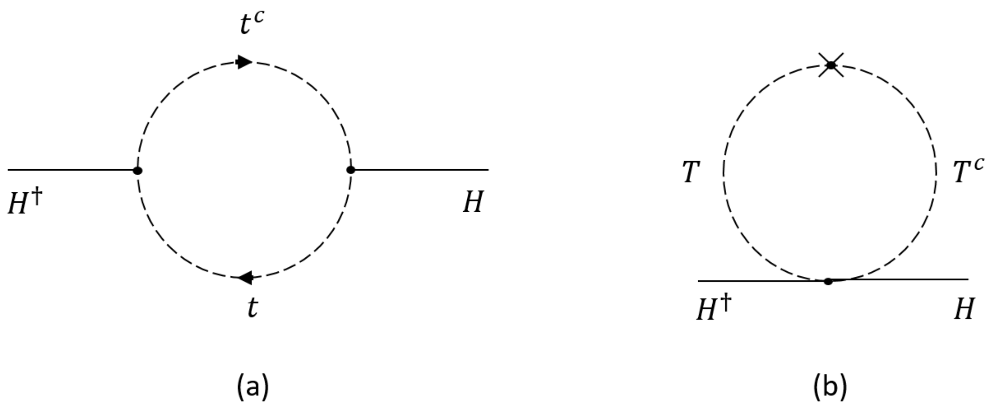


Figure 2.4: (a) The leading divergence of the top quark coupling, where the less contributing bottom quark b is omitted. (b) The compensating T loop to cancel the top quark coupling

the divergences previously discussed can be eliminated.

2.2.4 Back to electroweak symmetry breaking

After the discussion and clarification of the divergence elimination methods, it is indispensable to talk about the electroweak symmetry breaking again within the newly built frame, to give a smooth connection between the regular SM and our little higgs model as well as determining collective symmetry breaking energy scale f.

The remaining gauge coupling is Eq (2.25). By introducing a convenient parameterization

$$\begin{aligned}\Sigma_1 &= e^{\mathrm{i}K} e^{\mathrm{i}\Pi} \begin{pmatrix} \vec{0} \\ f \end{pmatrix} \\ \Sigma_2 &= e^{\mathrm{i}K} e^{-\mathrm{i}\Pi} \begin{pmatrix} \vec{0} \\ f \end{pmatrix}\end{aligned} \tag{2.28}$$

in which the coefficients are cancelled by choosing the unitary gauge. Then the expansion of $\Sigma_1^\dagger \Sigma_2$ reads

$$\begin{aligned}\Sigma_1^\dagger \Sigma_2 &= \begin{pmatrix} \vec{0} & f \end{pmatrix} e^{-2\mathrm{i}\Pi} \begin{pmatrix} \vec{0} \\ f \end{pmatrix} \\ &= f^2 - 2H^\dagger H + \cdots\end{aligned} \tag{2.29}$$

where we only consider the terms with only H. The equation will result in the divergence in the higgs mass as

$$|\Sigma_1^\dagger \Sigma_2|^2 = (f^2 - 2H^\dagger H + \cdots)(f^2 - 2H^\dagger H + \cdots) = f^4 - 4f^2 H^\dagger H + \cdots \quad (2.30)$$

Therefore, back to Eq (2.25), the mass term will be at the scale

$$\frac{g^4}{16\pi^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) f^2 \quad (2.31)$$

To establish the connection between our model and the SM electroweak symmetry breaking theory, such mass shall be at the scale $\sim M_w^2$ as discussed in Section 1.2. A more comprehensive analysis can yield a connection among the cutoff Λ, LH collective symmetry breaking scale f, electroweak scale M_w

$$\Lambda \sim 4\pi f \sim (4\pi)^2 M_w \quad (2.32)$$

by inserting loop factors. Considering the numerical electroweak scale, the f can be determined to have a value near TeV order. Consequently, we can summarize in our example $SU(3)/SU(2)$ will experience a collective symmetry breaking first up to the TeV scale, which will produce several NGBs that can acquire mass finally when the energy is down to the usual electroweak scale $\sim$ 100 GeV.

Finally the general little higgs mechanism can be introduced[8] after the analysis carried out throughout the section: The little higgs mechanism is the spontaneous symmetry breaking happening in some energy scale that produces several pesudo-Nambu-Goldstone bosons called 'little higgs', under which the symmetry is broken collectively. The 'collective' means that the symmetry is broken if there are two or more remaining non-vanishing gauge coupling constant. Otherwise, setting any of them to be zero can restore the symmetry and preserve the masslessness of the little higgs.

2.2.5 The simplest little higgs

After a theoretical derivation of the simplest $SU(3)/SU(2)$ collective symmetry breaking pattern, it is time to excavate into a realistic little higgs model under such set-up named **Simplest Little Higgs Model**[9], which is constructed with symmetry group $SU(3)_{color} \times SU(3)_{weak} \times U(1)_X$.
The SM particles are embedded in such symmetry group. For each, generation, the

abbreviated $(SU(3)_{color}, SU(3)_{weak})_{U(1)_X}$ representation reads

$$\begin{aligned} \Psi_Q &= (3,3)_{1/3} \qquad & \Psi_L &= (1,3)_{-1/3} \\ d^c &= (\bar{3},1)_{1/3} \qquad & e^c &= (1,1)_1 \\ 2u^c &= (\bar{3},1)_{-2/3} \qquad & n^c &= (1,1)_0 \end{aligned} \tag{2.33}$$

The triplet Ψ_Q, Ψ_L contain the usual SM left-handed quarks and leptons in the first two components and an additional third quark/lepton. One of the u^c, the d^c and e^c are the charge-conjugated (therefore left-handed) singlets, where the $\bar{3}$ stands for the anti-fundamental representation of $SU(3)$. The other u^c, along with the n^c are the third component Dirac partner from Ψ_Q, Ψ_L.
According to collective symmetry breaking, such $SU(3) \to SU(2)$ pattern requires two scalar field ϕ_1, ϕ_2 as $\phi_i = (1,3)_{-1/3}$. The Lagrangian terms have forms like

$$\begin{aligned} L_{kin} &\sim \Psi_Q^\dagger \not{D} \Psi_Q + \Psi_L^\dagger \not{D} \Psi_L + \cdots + (D_\mu \phi_1)^\dagger (D^\mu \phi_1) + (D_\mu \phi_2)^\dagger (D^\mu \phi_2) \\ L_{yuk} &\sim \lambda_1^u \phi_1^\dagger \Psi_Q u_1^c + \lambda_2^u \phi_2^\dagger \Psi_Q u_2^c + \frac{\lambda^d}{f} \phi_1 \phi_2 \Psi_Q d^c + \lambda^n \phi_1^\dagger \Psi_L n^c + \frac{\lambda^e}{f} \phi_1 \phi_2 \Psi_L e^c \\ L_{pot} &\sim \mu^2 \phi_1^\dagger \phi_2 \end{aligned} \tag{2.34}$$

With the parametrization of the VEV of the two scalar fields

$$\phi_1 = e^{\mathrm{i}\Theta \frac{f_2}{f_1}} \begin{pmatrix} \vec{0} \\ f_1 \end{pmatrix} \qquad \phi_2 = e^{-\mathrm{i}\Theta \frac{f_1}{f_2}} \begin{pmatrix} \vec{0} \\ f_2 \end{pmatrix} \qquad \Theta = \frac{\eta}{\sqrt{2} f} + \frac{1}{f} \begin{pmatrix} 0_{2\times 2} & h \\ h^\dagger & 0 \end{pmatrix} \tag{2.35}$$

with the energy scale $f^2 = f_1^2 + f_2^2$. The mass terms can be solved, alongside with the new gauge bosons $W'^{\pm}, W^{0\prime}, Z'$, several new fermions T, U, C and the scalar field η [9].

2.3 The Littlest Higgs L^2H

Having established the little higgs theory, it is time to look into more complicated higgs model. After the first appearance of 'big moose' model[8] utilizing the collective symmetry breaking into interpretation of electroweak symmetry breaking, a number of little higgs models based on different symmetry groups and versatile symmetry breaking patterns have been proposed. In this section, we will discuss one of the most economical, i.e. including relatively few numbers of postulates, and most attractive models, the **littlest higgs model**[10], which is also abbreviated as L^2H.

2.3.1 Basic setup

The littlest higgs model is embedded in a $SU(5)/SO(5)$ non-linear sigma model, where the original $SU(5)$ is spontaneously broken to $SO(5)$ at an energy scale $f \sim 1$ TeV, generating 14 NGBs and resolving 10 unbroken degrees of freedom. The model itself is considered as a symmetric two-index tensor

$$\square\square = \mathbf{15} \tag{2.36}$$

which means that the sigma field has the following transformation under $SU(5)$ matrix U

$$\Sigma \to U\Sigma U^T \tag{2.37}$$

In the symmetry breaking process, the VEV, or the condensate is chosen as

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & \mathbb{1} \\ 0 & 1 & 0 \\ \mathbb{1} & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \tag{2.38}$$

which is a 5×5 matrix, with two 2×2unit matrix at the up-right and down-left corners, in addition to a 1 in the center. For simplicity, we will use the former block expression in the following derivation.
According to Eq (2.37), the generators X^a in the coset $SU(5)/SO(5)$ and T^a from the unbroken subgroup $SO(5)$ have

$$T^a\Sigma_0 + \Sigma_0 T^{aT} = 0 \qquad (unbroken) \tag{2.39}$$
$$X^a\Sigma_0 - \Sigma_0 X^{aT} = 0 \qquad (broken) \tag{2.40}$$

The sigma field Σ, under both constraints: *i.* symmetric two-index tensor transforming as Eq (2.37) *ii.* in the broken generator basis as Eq (2.40), is generated as

$$\Sigma = e^{\frac{\mathrm{i}\Pi}{f}}\Sigma_0 e^{\frac{\mathrm{i}\Pi^T}{f}} = e^{\frac{2\mathrm{i}\Pi}{f}}\Sigma_0 \tag{2.41}$$

where the Π matrix is constructed from the 14 broken generators

$$\Pi = \sum_{a=1}^{14} \pi^a X^a \tag{2.42}$$

Again, the unitary condition $\Sigma^\dagger\Sigma = \mathbb{1}$ is regulated.

2.3.2 Gauge interaction

The higgs field is considered as a subgroup of the intact symmetry breaking, in which the gauged group $[SU(2)\times U(1)]^2$ from $SU(5)$ is chosen to be the original group that will be broken to the usual $SU(2)\times U(1)$ in electroweak process. Notice that in the process the collective symmetry breaking is utilized.
The generators in the $[SU(2)\times U(1)]^2$ are chosen with the form

$$Q_1^a = \begin{pmatrix} \frac{\sigma^a}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad Y_1 = \frac{1}{10}\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix} = \frac{1}{10}\mathrm{diag}(3,3,-2,-2,-2)$$
$$Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\sigma^a}{2} \end{pmatrix}, \qquad Y_2 = \frac{1}{10}\mathrm{diag}(2,2,2,-3,-3) \tag{2.43}$$

just as in Eq (2.17), the σ^a are the usual pauli matrices. Additional $U(1)$ charge is denoted as Y_i. And for the two Q_i^as, we are using the notation introduced in Eq (2.38).
The covariant derivative is, consequently,

$$D_\mu\Sigma = \partial_\mu\Sigma - \mathrm{i}\sum_{j=1}^{2}[g_j W_{j\mu}^a(Q_j^a\Sigma + \Sigma Q_j^{aT}) + g_j' B_{j\mu}(Y_j\Sigma + \Sigma Y_j)] \tag{2.44}$$

Eq (2.44) has such form since we have two different $[SU(2)\times U(1)]_i$ groups, while sigma field is, as the preceding notation, a symmetric two-index tensor.

Therefore, we can write the Lagrangian kinetic term

$$L_{kin} = \frac{f^2}{4}\mathrm{Tr}[(D_\mu\Sigma)^\dagger(D^\mu\Sigma)] \tag{2.45}$$

where the trace constant is chosen to be $\mathrm{Tr}[X^a X^b] = \delta^{ab}/2$ to normalize the lagrangian.
The higgs field symmetry breaking pattern can be identified qualitatively. The original product group $[SU(2)\times U(1)]^2$ from $SU(5)$ initially breaks to a single $SU(2)\times U(1)$, which can be recognized as the usual electroweak group, with a symmetry breaking energy scale f ∼1 TeV at tree level non-linear sigma model. Then, the SM higgs model participate in, transforming the $SU(2)\times U(1)\rightarrow U(1)_{EM}$, at a scale $v = 174$ GeV[3], which can be illustrated in to the following process

$$[SU(2)\times U(1)]^2 \xrightarrow{f\sim 1TeV} SU(2)_{weak}\times U(1)_Y \xrightarrow{v=174GeV} U(1)_{EM} \tag{2.46}$$

After a schematic illustration, the quantitative calculation can be carried out in Eq (2.44). The 14 broken NGBs from $SU(5)/SO(5)$ can be written in the representation of electroweak symmetry group $SU(2)\times U(1)$ as

$$\mathbf{1}_0 \oplus \mathbf{3}_0 \oplus \mathbf{2}_{\pm\frac{1}{2}} \oplus \mathbf{3}_{\pm 1} \tag{2.47}$$

where the number indicates the representation in weak gauge group, and the subscripts are the hypercharges from $U(1)$. In such representations, the fields are denoted respectively as η, ω, H and ϕ. Moreover, the transforming matrix in Eq (2.41) is explicitly[11]

$$\Pi = \begin{pmatrix} -\frac{\omega^0}{2}-\frac{\eta}{\sqrt{20}} & -\frac{\omega^+}{\sqrt{2}} & \frac{H^+}{\sqrt{2}} & -\mathrm{i}\phi^{++} & -\mathrm{i}\frac{\phi^+}{\sqrt{2}} \\ -\frac{\omega^-}{\sqrt{2}} & \frac{\omega^0}{2}-\frac{\eta}{\sqrt{20}} & \frac{H^0}{\sqrt{2}} & -\mathrm{i}\frac{\phi^+}{\sqrt{2}} & \frac{-\mathrm{i}\phi^0+\phi^0_P}{\sqrt{2}} \\ \frac{H^-}{\sqrt{2}} & \frac{H^{0*}}{\sqrt{2}} & \sqrt{\frac{4}{5}}\eta & \frac{H^+}{\sqrt{2}} & \frac{H^0}{\sqrt{2}} \\ -\mathrm{i}\phi^{--} & -\mathrm{i}\frac{\phi^-}{\sqrt{2}} & \frac{H^-}{\sqrt{2}} & -\frac{\omega^0}{2}-\frac{\eta}{\sqrt{20}} & -\frac{\omega^-}{\sqrt{2}} \\ -\mathrm{i}\frac{\phi^-}{\sqrt{2}} & \frac{\mathrm{i}\phi^0+\phi^0_P}{\sqrt{2}} & \frac{H^{0*}}{\sqrt{2}} & -\frac{\omega^+}{\sqrt{2}} & \frac{\omega^0}{2}-\frac{\eta}{\sqrt{20}} \end{pmatrix} \tag{2.48}$$

in which the superscripts are the electric charges that the field particle carries, and the coefficients are to make the normalization complete. However, during the symmetry breaking, the η and ω fields will be absorbed into the gauge bosons, thus the Eq (2.48) can be written alternatively as the off-diagonal block matrix

$$\Pi = \begin{pmatrix} 0 & \frac{H}{\sqrt{2}} & \phi \\ \frac{H^\dagger}{\sqrt{2}} & 0 & \frac{H^T}{\sqrt{2}} \\ \phi^\dagger & \frac{H^*}{\sqrt{2}} & 0 \end{pmatrix} \tag{2.49}$$

The gauge field interaction can be calculated given the Eq (2.41) and Eq (2.49), which will just follow the derivation in Section 2.2.1, but with 2 different $SU(2)$ gauge fields $W^a_{i\mu}$, and two $U(1)$ gauge field $B_{i\mu}$. Under the basis $(W^a_{1\mu}, W^a_{2\mu})$, the gauge coupling constants g_1, g_2 form a matrix considering the mass terms generated in the Lagrangian Eq (2.45)

$$\frac{f^2}{4}\begin{pmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{pmatrix} \tag{2.50}$$

The mass eigenstates can be solved once the eigenvalues and eigenvectors of the matrix are derived. We denote the two eigenstates as W^a_L, W^a_H, meaning the light and heavy mass eigenstate

$$\begin{aligned} W^a_L &= \sin\psi W^a_1 + \cos\psi W^a_2 \qquad M_{W_L} = 0 \\ W^a_H &= -\cos\psi W^a_1 + \sin\psi W^a_2 \quad M_{W_H} = \sqrt{g_1^2 + g_2^2}\frac{f}{2} \\ \sin\psi &= \frac{g_2}{\sqrt{g_1^2 + g_2^2}} \end{aligned} \tag{2.51}$$

where we define a mixing angle ψ representing the mixing between the two states. The $U(1)$ gauge field can be calculated with same method, but with a different normalizing constant. In $(B_{1\mu}, B_{2\mu})$ basis, the corresponding gauge coupling g'_1, g'_2 have

$$\frac{f^2}{20}\begin{pmatrix} g_1'^2 & -g'_1 g'_2 \\ -g'_1 g'_2 & g_2'^2 \end{pmatrix} \tag{2.52}$$

generating the following mass eigenstates B_L, B_H

$$\begin{aligned} B_L &= \sin\psi B_1 + \cos\psi B_2 \qquad M_{B_L} = 0 \\ B_H &= -\cos\psi B_1 + \sin\psi B_2 \quad M_{B_H} = \sqrt{g_1'^2 + g_2'^2}\frac{f}{\sqrt{20}} \\ \sin\psi' &= \frac{g'_2}{\sqrt{g_1'^2 + g_2'^2}} \end{aligned} \tag{2.53}$$

Similarly, the mixing angle between $B_{1\mu}, B_{2\mu}$ is defined to simplify the expression. Back to the electroweak subgroup $SU(2) \times U(1)$, the gauge coupling constant g, g'

are

$$g = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \qquad g' = \frac{g_1' g_2'}{\sqrt{g_1'^2 + g_2'^2}} \tag{2.54}$$

After the clarification of the gauge coupling, it is worthwhile to discuss the divergent terms. According to the collective symmetry breaking, such configurations should produce no quadratic divergence for the higgs field H in Eq (2.49), which can be shown explicitly when deriving the quadratic mass term in the kinetic Lagrangian that they should satisfy the following form

$$\frac{1}{4}(g_1 g_2 W_{1\mu}^a W_2^{\mu a} + g_1' g_2' B_{1\mu} B_2^\mu) H^\dagger H \tag{2.55}$$

such gauge interaction will result in a diagram similar to Fig 2.3. Notice that the direct multiplication $g_i^2 W_i^{a2} H^\dagger H$ (or $g_i'^2 B_i^2 H^\dagger H$) is in absence ,which eliminate the quadratically divergent term. The same idea can be shown more clearly in the mass eigenstates Eq (2.51), which gives

$$\begin{aligned}\frac{1}{4}[&g^2(W_{L\mu}^a W_L^{\mu a} - W_{H\mu}^a W_H^{\mu a} - 2\cot 2\psi W_{H\mu}^a W_L^{\mu a}) \\ &+ g'^2(B_{L\mu} B_L^\mu - B_{H\mu} B_H^\mu - 2\cot 2\psi' B_{H\mu} B_L^\mu)] H^\dagger H\end{aligned} \tag{2.56}$$

This expression illustrates the idea concisely, in which the quadratically divergent term in light gauge field cancel with that of the heavy one, as shown in Fig 2.5. For the $U(1)$ sector, the condition is the same. The only remaining divergent term comes from the logarithmic mixing one, therefore generating no pathological infinity. Nevertheless, such symmetry is not accidental, but inserted within the collective symmetry breaking.

On the other hand, the ϕ triplet defined in Eq (2.49) can not have such beautiful mechanism to help cancel its mass term, therefore will guarantee a mass term at the scale of $f \sim$ TeV.

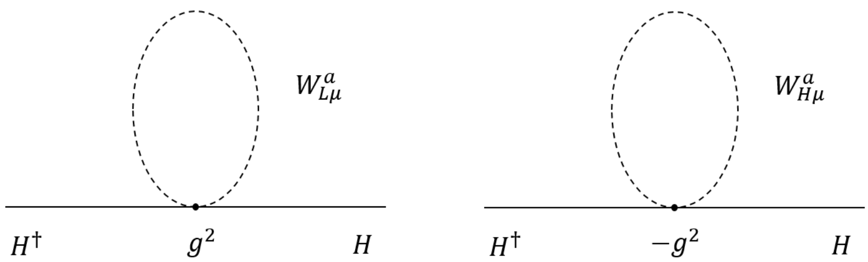


Figure 2.5: The one loop contribution from the $SU(2)$ sector to the Higgs boson with mass eigenstates.

2.3.3 Top quark sector

Thanks to the collective symmetry breaking, the disastrous quadratically divergent gauge interaction contribution is cancelled. However, the quark family yukawa coupling still give the challenges, where we can not simply choose a third quark to defend all the divergence, just like what we do in the $SU(3)/SU(2)$ simplest example. Therefore, a more complicated but useful method is to introduce a weak-singlet Weyl fermion pair U_L and compensating U_R with $+2e/3$ electric charge, which is coupled to the heaviest quark family $Q_{3L} = (t_{3L}, b_{3L})^T$ and corresponding t_{3R} as

$$L_{top} = -\frac{\lambda_1}{2} f \chi_L^\dagger \epsilon_{ijk} \epsilon_{mn} \Sigma_{jm} \Sigma_{kn} t_R - \lambda_2 f U_L^\dagger U_R + h.c. \tag{2.57}$$

where the χ is defined as the composition of usual SM quark and the Weyl fermion pair

$$\chi_L = \begin{pmatrix} \sigma_2 Q_{L3} \\ U_L \end{pmatrix} \tag{2.58}$$

and the vital importance is the range of the indices. The index $i, j, k = \{1, 2, 3\}$, and m, n runs $m, n = \{4, 5\}$, thus for the sigma field Σ, the indices indicate that the upper right 3×2 block is extracted. According to Eq (2.41) and Eq (2.49), this means that the ϕ and $H^T/\sqrt{2}$ blocks and their multiplication with other terms in higher order is included in the lagrangian yukawa term.

$$\begin{aligned} \Sigma =& \mathbb{1} + \frac{2i}{f} \begin{pmatrix} \phi & \frac{H}{\sqrt{2}} & 0 \\ \frac{H^\dagger}{\sqrt{2}} & 0 & \frac{H^T}{\sqrt{2}} \\ 0 & \frac{H^*}{\sqrt{2}} & \phi^\dagger \end{pmatrix} \\ & - \frac{2}{f^2} \begin{pmatrix} \frac{HH^\dagger}{2} & \frac{\phi H^*}{\sqrt{2}} & \frac{HH^\dagger}{2} + \phi\phi^\dagger \\ \frac{H^\dagger \phi}{\sqrt{2}} & \frac{H^\dagger H + H^T H^*}{2} & \frac{H^T \phi^\dagger}{\sqrt{2}} \\ \phi\phi^\dagger + \frac{H^* H^T}{2} & \frac{\phi^\dagger H}{\sqrt{2}} & \frac{H^* H^\dagger}{2} \end{pmatrix} \\ & + higher\ order \end{aligned} \tag{2.59}$$

The mass eigenstates can be calculated from the given expression of Σ, and as the preceding arguments show, the only considered part is the upper-right 2 columns.

$$\begin{aligned} t_L = t_{3L} \quad t_R &= \frac{\lambda_2 t_{3R} - \lambda_1 U_R}{\sqrt{\lambda_1^2 + \lambda_2^2}} \qquad M_{t_L} = 0 \\ T_L = U_L \quad t_R &= \frac{\lambda_2 t_{3R} + \lambda_1 U_R}{\sqrt{\lambda_1^2 + \lambda_2^2}} \qquad M_{t_L} = \sqrt{\lambda_1^2 + \lambda_2^2} f \end{aligned} \tag{2.60}$$

in which a third heavy top quark naturally shows up, similar to the discussion in Section 2.2.3. And the Lagrangian can be written in the mass eigenbasis

$$
\begin{aligned}
&\lambda_1(\sqrt{2}Q_{3L}\tilde{H} - \frac{1}{f}H^\dagger H U_L^\dagger)t_{3R} + h.c. \\
=&\lambda_t Q_{3L}\tilde{H}t_R + \lambda_T Q_{3L}\tilde{H}T_R - \frac{1}{\sqrt{2}f}H^\dagger H T_L^\dagger(\lambda_T T_R + \lambda_t t_R) + h.c.
\end{aligned}
\tag{2.61}
$$

in which we define $\tilde{H} = \mathrm{i}\sigma^2 H$ and

$$
\lambda_t = \frac{\sqrt{2}\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}, \qquad \lambda_T = \frac{\sqrt{2}\lambda_1^2}{\sqrt{\lambda_1^2+\lambda_2^2}} \tag{2.62}
$$

The first term in Eq (2.61) is just the SM top quark coupling, with the λ_t interpreted as top quark yukawa coupling constant. A more precise derivation of all the terms for the top quark coupling can be found in Ref [12].
The collective symmetry breaking pattern can be found in such designation. Just as the statement above, 'any one vanishing coupling value will result in the masslessness of little higgs.', if the $\lambda_1 = 0$, the Eq (2.57) has no sigma field component, meaning it is invariant to the top quark section. Such pattern is also embedded in Eq (2.62), where the coupling constants become 0. On the other hand, if λ_2 is set to 0, the only remaining λ_1 part will not break the $SU(3)$ symmetry, therefore ensuring the little higgs as NGBs. Consequently, on either case, the little higgs remains massless. And the only contribution shall be the one involving both terms, generating logarithmic divergence as discussed repeatedly in preceding section. More explicitly, the mass

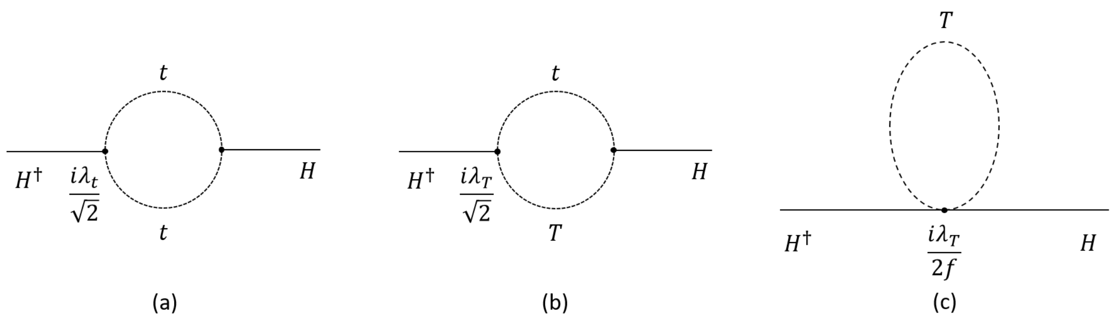


Figure 2.6: The one-loop correction to the higgs mass in the top quark sector, with the vertex value explicitly depicted.

eigenstates generate the one-loop contribution as the 3 diagrams shown in Fig 2.6.

The expressions for the Feynmann diagrams are respectively[12]

$$
\begin{aligned}
(a) &\to -6\lambda_t^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2} \\
(b) &\to -6\lambda_T^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2-M_T^2} \\
(c) &\to 6\frac{\sqrt{2}\lambda_T^2}{f} \int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{M_T}{k^2-M_T^2}
\end{aligned}
\tag{2.63}
$$

Such quadratically divergent term can be cancelled since in Eq (2.60) the M_T has been derived as

$$
M_T = \sqrt{\lambda_1^2+\lambda_2^2}f = \frac{\lambda_t^2+\lambda_T^2}{\sqrt{2}\lambda_T}f \tag{2.64}
$$

Therefore, the vanishing quadratically divergent term from top quark sector is demonstrated. Moreover, the mass of the predicted new heavy top quark can be measured in the future collider experiment, which will be mainly discussed in the following chapter.

2.3.4 Electroweak symmetry breaking under L^2H

In the preceding section 2.3.2, the gauge interaction is analysed and the conclusion that, the higgs field has no quadratically divergent mass term, but the ϕ triplet term can induce one, has been derived. In this section, how the detailed interaction of such terms takes place from f down to electroweak scale M_W is discussed. First, the Coleman-Weinberg potential should be considered for ϕ and H. The one-loop correction, as mentioned in Appendix B, gives a quadratically divergent term

$$
V_g^{(quad)} = \frac{\Lambda^2}{16\pi^2}\mathrm{Tr}M_V^2(\Sigma) \tag{2.65}
$$

where the M_V^2 is the gauge boson mass matrix that can be calculated from the kinetic term we have derived in Eq (2.45), having a form like

$$
(M_V^2)^{ab} = \sum_j g_j^2\{\mathrm{Tr}[(Q_j^a\Sigma+\Sigma Q_j^{aT})^\dagger(Q_j^b\Sigma+\Sigma Q_j^{bT})]+g_j'^2[(Y_j\Sigma+\Sigma Y_j^T)^\dagger(Y_j\Sigma+\Sigma Y_j^T)]^{ab}\} \tag{2.66}
$$

Using Eq (2.32), where $\Lambda \sim 4\pi f$, the potential term can be expressed

$$
V_g^{(quad)} = af^4\sum_j\left[g_j^2\sum_b \mathrm{Tr}(Q_j^b\Sigma Q_j^{b*}\Sigma^*) + g_j'^2\mathrm{Tr}(Y_j\Sigma Y_j^*\Sigma^*)\right] \tag{2.67}
$$

where the coefficient a is a constant gathering all non-important term and has a scale of order 1, just as the c and c' constant in Eq (2.20) and Eq (2.21). Expanding the Eq (2.67) to quadratic term in ϕ and quartic term in H results

$$
\begin{aligned}
V_g^{(quad)} =& a(g_1^2+g_1'^2)f^2|\phi_{ij}+\frac{\mathrm{i}}{4f}(H_iH_j+H_jH_i)|^2 \\
&+a(g_2^2+g_2'^2)f^2|\phi_{ij}-\frac{\mathrm{i}}{4f}(H_iH_j+H_jH_i)|^2
\end{aligned} \tag{2.68}
$$

On the other hand, the quadratically divergent term from the top quark sector to Coleman-Weinberg potential is

$$
V_t^{(quad)} = -a'\lambda_1^2\epsilon^{wx}\epsilon_{yz}\epsilon^{ijk}\epsilon_{kmn}\Sigma_{iw}\Sigma_{jx}\Sigma^{*my}\Sigma^{*nz} + h.c. \tag{2.69}
$$

Again, in Eq (2.69), the $i,j,k,m,n=\{1,2,3\}$ and $w,x,y,z=\{4,5\}$, making sure that the ϕ and $H^T/\sqrt{2}$ are included.
Further expansion of H and ϕ results in

$$
V_t^{(quad)} = -a'\lambda_1^2f^2|\phi_{ij}+\frac{\mathrm{i}}{4f}(H_iH_j+H_jH_i)|^2 \tag{2.70}
$$

The results from Eq (2.68) and Eq (2.70) show that the quadratically divergent term of H are cancelled, as indicated from collective symmetry breaking. However, the mass term of ϕ remains as

$$
M_\phi^2 = [a(g_1^2+g_1'^2+g_2^2+g_2'^2)-a'\lambda_1^2]f^2 \tag{2.71}
$$

At energy below the triplet mass, the triplet ϕ should be integrated out from the Lagrangian, giving a quartic potential to the Higgs field as $\lambda|H^\dagger H|^2$ where [10]

$$
\lambda = a\frac{(g_1^2+g_1'^2-\frac{a'\lambda_1^2}{a})(g_2^2+g_2'^2)}{g_1^2+g_1'^2+g_2^2+g_2'^2-\frac{a'\lambda_1^2}{a}} \tag{2.72}
$$

The remaining logarithmically divergent contribution in Coleman-Weinberg potential comes from

$$
V_g^{(log)} = \frac{3}{64\pi^2}\mathrm{Tr}M_V^4(\Sigma)\log\frac{M_V^2(\Sigma)}{\Lambda^2} \tag{2.73}
$$

Expansion of Σ can be implemented into the equation to yield the mass parameter μ_g^2, where the Eq (2.51) is used to condensate the expression to the heavy gauge field W_H, B_H

$$
\mu_g^2 = \frac{3}{64\pi^2}\left(3g^2M_{W_H}^2\log\frac{\Lambda^2}{M_{W_H}^2}+g'^2M_{B_H}^2\log\frac{\Lambda^2}{M_{B_H}^2}\right) \tag{2.74}
$$

Additionally, the triplet ϕ also exerts a logarithmically divergent contribution to the higgs mass from Eq (2.67) as

$$\mu_s^2 = \frac{\lambda}{16\pi^2} M_\phi^2 \log \frac{\Lambda^2}{M_\phi^2} \tag{2.75}$$

where the λ is defined in Eq (2.72).
For the top quark sector, the loop contribution has a logarithmic divergence as

$$V_t^{(log)} = -\frac{3}{16\pi^2}\mathrm{Tr}[M_t(\Sigma)M_t^\dagger(\Sigma)]^2 \log \frac{M_t(\Sigma)M_t^\dagger(\Sigma)}{\Lambda^2} \tag{2.76}$$

resulting in a negative contribution to the Higgs mass

$$\mu_t^2 = -\frac{3\lambda_t^2 M_T^2}{8\pi^2} \log \frac{\Lambda^2}{M_T^2} \tag{2.77}$$

Therefore, from Eq (2.74), (2.75), (2.77), the dependence of the one-loop contribution to Higgs sector is on the four new particles with mass at f scale, the gauge bosons W_H, B_H, the heavy top quark T and a triplet scalar field ϕ.
After the electroweak symmetry breaking, the littlest higgs model should break into the ordinary life – the SM predicted world. Nevertheless, there are still traces to some minor effects.
Firstly, the Higgs field in L^2H will be decomposed into

$$H = \begin{pmatrix} \pi^+ \\ v + \frac{h+\mathrm{i}\pi^0}{\sqrt{2}} \end{pmatrix} \tag{2.78}$$

where the $v = 174$ GeV is the usual electroweak parameter, and h is the physical higgs field. The $\pi^\pm$ and π^0 field will be absorbed, along with small amount of $\omega^\pm/\omega^0$ at order v^2/f^2 and $\phi^\pm/\phi^0$ at order v'^2/v^2 (v' is defined below in Eq (2.79))[13], by the SM $W^\pm$ and Z^0 bosons. Therefore, the mass of $W^\pm$ and Z^0 will receive corrections at order v^2/f^2, which will be interpreted in Section 3.
Moreover, the Coleman-Weinberg potential will contain $H^\dagger \phi H$ term, generating a tadpole diagram for ϕ after the Higgs acquires mass at the low energy v, which will therefore violate the custodial symmetry $SU(2)$

$$v' = \langle \frac{-\mathrm{i}\phi^0 + \phi_P^0}{\sqrt{2}} \rangle = -\mathrm{i}\frac{v^2}{4M_\phi} = -\mathrm{i}\frac{v^2 f}{4M_\phi^2}[a(g_1^2 + g_1'^2 + g_2^2 + g_2'^2) - a'\lambda_1^2] \tag{2.79}$$

which shall also generate an anomaly at microscopic view.

2.4 Alternative models based on L^2H

In the preceding section the basic characteristics of the littlest higgs model has been introduced and derived. Serving as a supplementation, some alternative little higgs models are presented, with modification of the original symmetry group or implementation of new mechanism to the existing model. Such modification will exert effects on the predicted behavior of certain particle and the energy level of f, which will be explored in the following subsections.

2.4.1 $SU(6)/Sp(6)$ model

The $SU(6)/Sp(6)$ little higgs model is a variation to the Littlest higgs model, where the original symmetry group is now $SU(6)$ and break down to the compact symplectic group $Sp(6)$. There are $35 - 21 = 14$ NGBs, the same number as in the Littlest higgs case. However, the VEV is an antisymmetric 6-dimensional matrix

$$\Sigma_0 = \begin{pmatrix} 0 & -\mathbb{1}_{3\times3} \\ \mathbb{1}_{3\times3} & 0 \end{pmatrix} \tag{2.80}$$

The gauge field symmetry of higgs field is the same as the preceding L^2H, an $[SU(2)\times U(1)]^2$ spontaneously broken to the usual $SU(2)\times U(1)$. The 14 NGBs split 4 of them to be absorbed in such symmetry breaking, similar to the fate of η, ω in L^2H model. However, there are no triplet generation like the ϕ in Eq (2.48). Instead, there are two Higgs doublets along with one complex singlet to divide the 10 NGBs as

$$\mathbf{2}_{\frac{1}{2}} \oplus \mathbf{2}_{-\frac{1}{2}} \oplus \mathbf{1}_0 \tag{2.81}$$

In this model, in consequence of having the almost identical symmetry breaking pattern with L^2H, there will also be the heavy gauge field bosons, no triplet but 2 composite higgs doublets, and some heavy quarks, with the same derivation methods. Nevertheless, the value of the bosons are shifted as a result of different symmetry groups, which can be calculated as[14]

$$M_{W_H} = \sqrt{g_1^2 + g_2^2}\frac{f}{2} \qquad M_{B_H} = \sqrt{g_1'^2 + g_2'^2}\frac{f}{2\sqrt{2}} \tag{2.82}$$

where the SM gauge coupling g, g' are defined just the same as Eq (2.54). Therefore, it is obvious that the $SU(6)/Sp(6)$ model has the same structure as the L^2H, except that it eliminates the divergent triplet ϕ. Such elimination, on the other hand, affects the mass of the heavy gauge bosons, as well as the energy scale f, which enables us to choose a relatively smaller value of f, though still at the level of TeV.

2.4.2 L^2H with T parity

Besides the $SU(6)/Sp(6)$ model, the elimination of triplet can be carried out just within the original L^2H, but with a further symmetry called T parity, which is similar to the R parity in Minimal Supersymmetric Standard Model (MSSM). The T parity will be implemented into the product groups, namely the $[SU(2)\times U(1)]^2$, which can be treated as the product of two $SU(2)\times U(1)$, denoted as $G_i = [SU(2)\times U(1)]_i$. Such modified L^2H is called the **Littlest Higgs with T Parity (L^2HT)**. To better describe the geometry of T-parity, the whole $SU(5)/SO(5)$ symmetry breaking should be considered. Remember in Section 2.3.1 we denote the unbroken $SO(5)$ generators as T^a, and the broken $SU(5)/SO(5)$ coset generators as X^a. In T parity, such generators satisfy

$$[T^a, T^b] \sim T^c \qquad [T^a, X^b] \sim X^c \qquad [X^a, X^b] \sim X^c \tag{2.83}$$

where the Lie brackets are just qualitatively given, only to illustrate the idea. For the symmetric two-index tensor, such Lie algebra will generate an automorphism with negative parity on the broken generators

$$\begin{aligned} T^a &\to T^a \\ X^a &\to -X^a \end{aligned} \tag{2.84}$$

which can be concisely written as the relation of generators $\mathcal{T}^a$ of the whole $SU(5)$ group

$$\mathcal{T}^a \to \Sigma_0(\mathcal{T}^a)^T\Sigma_0 \tag{2.85}$$

with the Σ_0 the usual VEV defined in Eq (2.38). The Σ field and Π will transform as

$$\Sigma \to \Sigma' = \Sigma_0\Omega\Sigma^\dagger\Omega\Sigma_0 \qquad \Pi \to -\Omega\Pi\Omega \tag{2.86}$$

where

$$\Omega = \mathrm{diag}(1, 1, -1, 1, 1) \tag{2.87}$$

Under such preliminaries, the higgs doublet H is T parity even while the triplet ϕ is T odd. Therefore, the gauge interaction term as discussed in Section 2.3.2 can not inherit the term like $H^\dagger\phi H$, as a result of T-parity conservation. By this means, the quadratically divergent term for ϕ is eliminated therefore acquiring no mass term. The gauge bosons from either G_1, G_2 under T parity can be transformed into the other group via

$$W^a_{1\mu} \overset{T}{\leftrightarrow} W^a_{2\mu} \qquad B_{1\mu} \overset{T}{\leftrightarrow} B_{2\mu} \tag{2.88}$$

Eq (2.88) leaves powerful constraints on the gauge coupling constant g_1, g_2 as well as the primed g_1', g_2', since the transformation change no numerical values. Therefore, the two coupling constant should have the same value. In consequence, the mixing angle is forced to be

$$\begin{aligned} \sin\psi &= \frac{\pi}{4} \qquad \sin\psi' = \frac{\pi}{4} \\ g_1 = g_2 &= \sqrt{2}g \qquad g_1' = g_2' = \sqrt{2}g' \end{aligned} \tag{2.89}$$

and the heavy gauge bosons are now

$$M_{W_H} = gf \qquad M_{B_H} = \frac{g'f}{\sqrt{5}} \tag{2.90}$$

For fermion section, the L^2HT suffers from some extra mechanism, since there should be additive terms to ensure the T-odd fermions acquires mass correctly.
For the first two generation fermions, the usual SM fermion doublet of generation ψ_1, ψ_2 can be embedded into the $SU(5)$ representation

$$\Psi_1 = \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} 0 \\ \psi_2 \end{pmatrix} \tag{2.91}$$

where the 0 stands for a 3-dimensional zero vector. The $SU(5)$ spinor transforms as

$$\Psi_1 \to U^*\Psi_1, \qquad \Psi_2 \to U\Psi_2 \qquad U \in SU(5) \tag{2.92}$$

To give mass terms for the T-odd terms, the mirror fermions ψ_c, χ_c, which is defined to preserve the parity, are included and represented as a $SO(5)$ right-handed multiplets Ψ_c

$$\Psi_c = \begin{pmatrix} \tilde{\psi}_c \\ \chi_c \\ \psi_c \end{pmatrix} \tag{2.93}$$

The Ψ_c transforms under $SO(5)$ as $\Psi_c \to V\Psi_c$, where the transforming tensor V itself is defined through the transformation of the matrix $\xi = \exp(\mathrm{i}\Pi/f)$ from the non-linear sigma field $\Sigma = \exp(2\mathrm{i}\Pi/f)\Sigma_0$

$$\xi \to V\xi\Sigma_0 U^T \Sigma_0 = U\xi V^T \tag{2.94}$$

After defining the spinors, the T-parity is exerted for

$$\Psi_1 \overset{T}{\longleftrightarrow} \Sigma_0\Psi_2, \qquad \Psi_c \overset{T}{\longleftrightarrow} -\Psi_c \tag{2.95}$$

Therefore, the yukawa fermion sector in the Lagrangian for the down quark family is

$$\frac{\mathrm{i}\lambda_d f}{2\sqrt{2}}\epsilon_{ij}\epsilon_{xyz}[X\bar{\Psi'}_x\Sigma_{iy}\Sigma_{jz} - X'(\bar{\Psi}\Sigma_0)_x\Sigma'_{iy}\Sigma'_{jz}]d_R \tag{2.96}$$

in which the primed parameters are the correspondence under T-parity. X is a scalar for gauge invariance, transforming as a $SU(2)_{1,2}$ singlet, and has a $U(1)_{1,2}$ charge respectively as $(Y_1, Y_2) = (1/10, -1/10)$. The value is chosen to be $X = (\Sigma_{33})^{-1/4}$. Such T-parity invariant Lagrangian will result in such mass term

$$L_\kappa = -\kappa f(\bar{\Psi}_2\xi\Psi_c + \bar{\Psi}_2\Sigma_0\Omega\xi^\dagger\Omega\Psi_c) + h.c. \tag{2.97}$$

For top quark family, the situation is a little bit more complicated, since there will be an additional $T_{L/R}$ heavy top quark. The representation in $SU(5)$ is now

$$\Psi_{t1} = \begin{pmatrix} \sigma^2 Q^1_{3L} \\ T_{1L} \\ 0 \end{pmatrix} \qquad \Psi_{t2} = \begin{pmatrix} 0 \\ T_{2L} \\ \sigma^2 Q^2_{3L} \end{pmatrix} \tag{2.98}$$

where the $Q^i_{3L} = (t^i_{3L}, b^i_{3L})$ is just the top quark family in SM, except that there are now 2 components in the representation. Under T-parity, they transform as

$$\Psi_{t1} \overset{T}{\leftrightarrow} \Psi_{t2}, \qquad T_{1R} \overset{T}{\leftrightarrow} T_{1R}, \qquad T_{2R} \overset{T}{\leftrightarrow} -T_{2R} \tag{2.99}$$

The corresponding yukawa term is just similar to Eq (2.96), except a additional heavy top quark sector

$$\frac{\lambda_1 f}{2\sqrt{2}}\epsilon_{ijk}\epsilon_{xy}[\bar{\Psi'}_i\Sigma_{jx}\Sigma_{ky} - (\bar{\Psi}\Sigma_0)_i\Sigma'_{jx}\Sigma'_{ky}]t_R + \lambda_2 f(\bar{T}_{1L}T_{1R} + \bar{T}_{2L}T_{2R}) \tag{2.100}$$

where again as Eq (2.57), the $\{i,j,k\} = \{1,2,3\}$ and $\{x,y\} = \{4,5\}$. The T-parity eigenstate is the superposition of the two sectors defined in Eq (2.98)

$$q_{L\pm} = \frac{Q^1_{3L} \mp Q^2_{3L}}{\sqrt{2}} \qquad T_{L\pm} = \frac{T_{1L} \mp T_{2L}}{\sqrt{2}} \qquad T_{R\pm} = \frac{T_{1R} \mp T_{2R}}{\sqrt{2}} \tag{2.101}$$

where the subscript $\pm$ indicate the T odd/even property of the parameter, with $+$ stands for even and $-$ for odd. The Lagrangian can accordingly be transformed into the eigenstates as

$$\lambda_1 f\left(\frac{s_\Sigma}{\sqrt{2}}\bar{t}_{L+}t_{R+} + \frac{1+c^2_\Sigma}{2}\bar{T}_{L+}t_{R+}\right) + \lambda_2 f(\bar{T}_{L+}T_{R+} + \bar{T}_{L-}T_{R-}) \tag{2.102}$$

with $s_\Sigma = \sin\frac{\sqrt{2}h}{f}, c_\Sigma = \cos\frac{\sqrt{2}h}{f}$. The T-odd heavy top quark is straightforward

$$M_{T-} = \lambda_2 f \tag{2.103}$$

Solving the λ_1 part with eigenbasis $(\bar{t}_{L+}, \bar{T}_{L+})\mathcal{M}(t_{R+}, T_{R+})^T$, with matrix $\mathcal{M}$

$$\mathcal{M} = \begin{pmatrix} \frac{\lambda_1 f}{\sqrt{2}}\sin\frac{\sqrt{2}h}{f} & 0 \\ \lambda_1 f\cos^2\frac{\sqrt{2}h}{f} & \lambda_2 f \end{pmatrix} \tag{2.104}$$

the mass eigenvalue is, to the first order of v/f

$$M_{t+} = M_{tSM} = \frac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}v, \qquad M_{T+} = \sqrt{\lambda_1^2+\lambda_2^2}f \tag{2.105}$$

Chapter 3

Interpretation: M_W Anomaly with Approximate LH

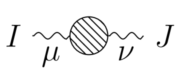
In the preceding sections the SM higgs mechanism and the little higgs model has been introduced and explained. In this chapter, the Little higgs model will be investigated to predict the effects onto the energy level f, combined with the recent W boson mass anomaly. In the beginning, the electroweak observables are introduced to give a convenient parameterization of the SM elementary variables. Then, the M_W anomaly is included to the specific little higgs model, giving a corrected prediction to the particles and symmetry breaking energy interpreted by the model.

3.1 Electroweak Precision Observable (EWPO) and oblique parameters

In SM, there are electroweak parameters defined to preserve the key characteristics of $SU(2) \times U(1)$ structure, including the W boson mass M_W, the weak mixing angle $\sin^2 \theta_W$, which contains the dominant information to the energy level, the branch ratio of specific reaction etc. Therefore, an accurate measurement, or in other word, the precision measurement, of such parameters are of vital importance to the high energy physics, not only to fortify the existing SM to higher precision, but also provide more powerful constrains on further BSM models, in reverse refining the prospective models. Such parameters, either defined within SM, or some promising observables predicted by BSM models, eg. the anomalous μ magnetic moment $g_\mu - 2$, are classified as **Electroweak Precision Observables**, or EWPO in some documentation.

In such measurements, with the developments of new detecting technology and higher energy accelerator, as well as measurement on wider energy spectrum, the achieved precision of the data can be at such high level that the loop-corrections, or radiative correction shall be included to provide an authentic theoretical prediction value. Recalling the procedures implemented on M_W in Section 1.2, the idea is just the same: to include higher-order interference comparing with the data extracted from experiments. Focusing on such correction, the **Oblique parameter**, or Peskin-Takeuchi S,T,U parameter named after who proposed them[15], are defined to provide a convenient expression for the radiative correction. Moreover, the effects of new physics, which shall have tiny but non-zero corrections to the SM observables, can also be expressed in the oblique parameters to have a realistic correspondence.

The oblique parameters are based on the vacuum polarization effects. The vacuum polarization amplitude involving two gauge bosons I, J can be denoted as



$$= \mathrm{i}\Pi^{\mu\nu}_{IJ}(q) \tag{3.1}$$

where the μ, ν are the subscript of the component of the gauge fields, and the q denotes the momentum. Examples of some electroweak gauge bosons are shown in Fig 3.1. For $I, J = \gamma$, the masslessness of photon requires the $\Pi^{\mu\nu}_{\gamma\gamma}(0) = 0$ to preserve its transverse property. However, if the gauge bosons are massive, like W, Z, such requirements will naturally lift off, and the generalized decomposition of the amplitude is

$$\Pi^{\mu\nu}_{IJ}(q) = \Pi_{IJ}(q^2)g^{\mu\nu} - \Delta(q^2)q^{\mu}q^{\nu} \tag{3.2}$$

The Π_{IJ} is the so-called **one particle irreducible propagator**, or abbreviated as **1PI**. The word 'irreducible' means that the diagram can not decompose into other one particle propagator, i.e. there is no other one particle intermediate propagator (or a line) in the diagram.

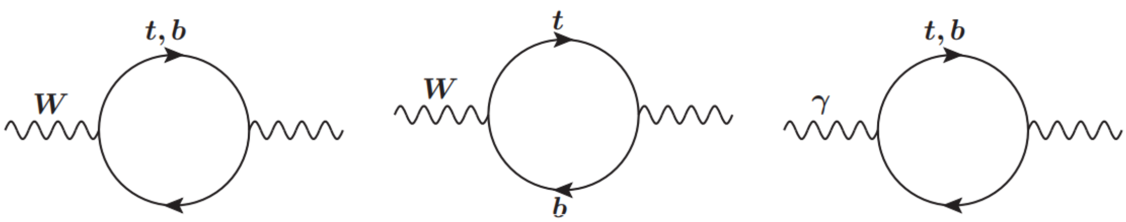


Figure 3.1: Examples for the vacuum polarization diagram of W and γ propagator. Again, here the heaviest t, b channel is only considered

Such vacuum polarization amplitude can provide corrections to the mass of the gauge bosons, like for the W boson

$$M_W^2 = \frac{g^2v^2}{4} + \Pi_{WW}(M_W^2) \tag{3.3}$$

which is similar to Eq (1.35), generating new mass corrections to W boson.
Such effects on kinetic terms of the SM Lagrangian on the usual gauge bosons can be denoted as

$$\Delta L = -\frac{1}{2}W_{3\mu}\Pi_{W_3W_3}(q^2)W_3^{\mu} - W_{3\mu}\Pi_{W_3B}(q^2)B^{\mu} - \frac{1}{2}B_{\mu}\Pi_{BB}(q^2)B^{\mu} - W_{+\mu}\Pi_{W_+W_-}(q^2)W_-^{\mu} \tag{3.4}$$

where the definition of W_+, W_- Eq (1.17)are utilized. In addition, Such expressions of Π_{IJ} are sometimes written as Π_{33}, Π_{3B}, Π_{+-} to abbreviate the W boson field.
Since the vacuum polarization is a loop effect, the 4 momentum q^2 can be assumed to be small, compared with the divided tree-level effects, therefore expanding the Π as

$$\Pi_{IJ}(q^2) = \Pi_{IJ}(0) + q^2\Pi'_{IJ}(0) + \frac{q^4}{2}\Pi''_{IJ}(0) + \mathcal{O}(q^6) \tag{3.5}$$

Up to the q^4 terms, consequently the Eq (3.4) contains 12 factors, or more precisely form factors, among which there are 3 constrained by the experimental outcome, $\Pi'_{W_+W_-}(0) = \Pi'_{BB} = 1$ as the normalization to the gauge coupling, and $\Pi_{W_+W_-} = -M_W^2$ as the renormalized mass of W boson. 2 further constraints shows up as the masslessness of γ requires $\Pi_{Z\gamma} = \Pi_{\gamma\gamma} = 0$, which can be decomposed into the basis of $\{W_{3\mu}, B_{\mu}\}$. The remaining 7 of them are undetermined parameters, denoted as $\hat{S}, \hat{T}, \hat{U}, V, W, X, Y$. A comprehensive definition and derivation about the expression of these oblique parameters is included in Ref [16] (without normalization). In the scope of this dissertation, the only important parameters are the $\hat{S}, \hat{T}, Y, W$, with the value given in Table 3.1 the first column. Notice that the hatted $\hat{S}, \hat{T}$ are correlated to the original S, T[15] by

$$S = \frac{4\sin^2\theta_W\hat{S}}{\alpha} \approx 119\hat{S} \qquad T = \frac{\hat{T}}{\alpha} \approx 129\hat{T} \tag{3.6}$$

where the α is the usual electroweak fine structure constant, but with a value between $1/129 \sim 1/127$ as a result of much higher energy level (above electroweak scale) to the regular EM field energy level.

On the other hand, such form factors have a close relation with the Standard Model Effective Field Theory (SMEFT), under which the dimension-6 operators, non-renormalizable but is a convenient way to parameterize the universal new physics, are implemented into the SM Lagrangian. The related dimension-6

Dimensionless form factor	Operator
$(g'/g)\hat{S} = \Pi'_{W_3B}(0)$	$\mathcal{O}_{WB} = (H^\dagger \tau^a H) W^a_{\mu\nu} B_{\mu\nu}$
$M_W^2 \hat{T} = \Pi_{W_3W_3}(0) - \Pi_{W_+W_-}(0)$	$\mathcal{O}_H = \lvert H^\dagger D_\mu H \rvert^2$
$2M_W^{-2} Y = \Pi''_{BB}(0)$	$\mathcal{O}_{BB} = (\partial_\rho B_{\mu\nu})^2/2 = J_B^2$
$2M_W^{-2} W = \Pi''_{W_3W_3}(0)$	$\mathcal{O}_{WW} = (D_\rho W^a_{\mu\nu})^2/2 = J_W^2$

Table 3.1: The dimensionless oblique parameters or form factors and the corresponding $SU(2)_L$-invariant universal dimension-6 operators[17]

operators are

$$L = L_{SM} + \frac{1}{v^2}(c_{WB}\mathcal{O}_{WB} + c_H\mathcal{O}_H + c_{BB}\mathcal{O}_{BB} + c_{WW}\mathcal{O}_{WW}) \tag{3.7}$$

with correspondence

$$\hat{S} = \frac{2\cos\theta_W}{\sin\theta_W} c_{WB}, \quad \hat{T} = -c_H, \quad Y = -g^2 c_{BB}, \quad W = -g^2 c_{WW} \tag{3.8}$$

Back to the SM will require all coefficients to be 0, which simultaneously neutralises all the oblique parameters.

3.2 Fitting M_W with Littlest Higgs Model

Using the convenient notation of oblique parameters, the fitting of recent M_W anomaly into L^2H can be implemented. Recall Eq (2.51) and Eq (2.53), where the mass of two extra heavy gauge bosons and two corresponding mixing angle ψ, ψ' are defined, which can also be written as

$$\sin\psi = \frac{g}{g_1}, \qquad \cos\psi = \frac{g}{g_2}, \qquad \sin\psi' = \frac{g'}{g'_1}, \qquad \cos\psi' = \frac{g'}{g'_2} \tag{3.9}$$

The universal corrections via oblique parameters are[18]

$$\begin{aligned}
\hat{S} &= \frac{2M_W^2}{g^2 f^2}\left(\cos^2\psi + \frac{5\cos^2\theta_W}{\sin^2\theta_W}\cos^2\psi'\right) \\
\hat{T} &= \frac{5M_W^2}{g^2 f^2} \\
W &= \frac{4M_W^2}{g^2 f^2}\cos^4\psi \\
Y &= \frac{20M_W^2}{g'^2 f^2}\cos^4\psi'
\end{aligned} \tag{3.10}$$

where the triplet ϕ effects on $\hat{T}$ are omitted. The anomaly $\hat{T} \approx 10^{-3}$ will result in the energy scale $f \sim 9$ TeV, as shown in Fig 3.2a[17]. According to Eq (3.9),the

green area compatible with the littlest higgs model with small $\cos\psi$ and $\cos\psi'$ implies that the coupling g_2, g_2' shall be much larger than the g, g' value, therefore eliminating the W, Y corrections.
For the Littlest Higgs with T parity, the result for the oblique parameters are nearly the same. Considering the same mass of heavy boson, the plausible oblique parameters are

$$\begin{aligned} \hat{S} &= \frac{2M_W^2}{g^2 f^2}\left(\cos^2\psi + \frac{\cos^2\theta_W}{\sin^2\theta_W}\cos^2\psi'\right) \\ \hat{T} &= \frac{5M_W^2}{g^2 f^2} \\ W &= \frac{4M_W^2}{g^2 f^2}\cos^4\psi \\ Y &= \frac{4M_W^2}{g'^2 f^2}\cos^4\psi' \end{aligned} \tag{3.11}$$

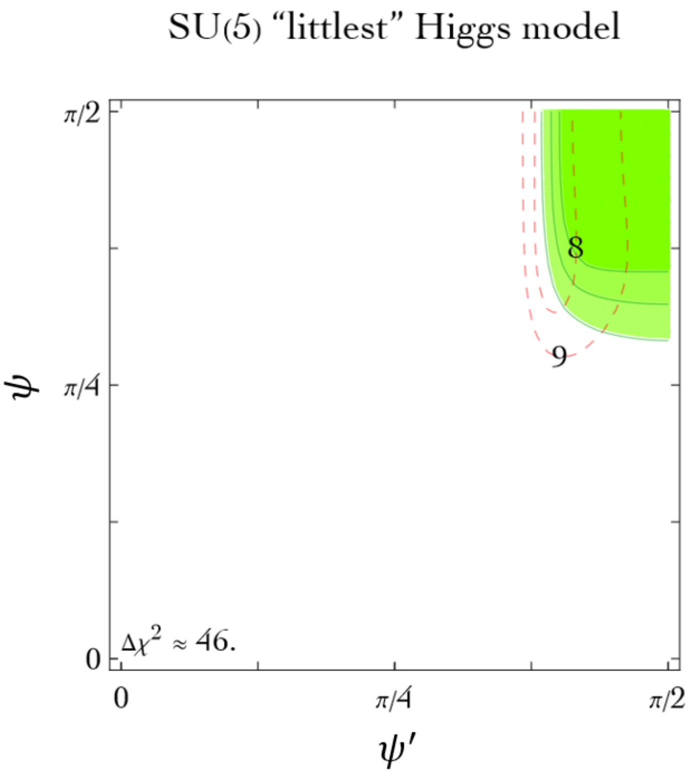
Nevertheless, such derivation is not available since as shown in Eq (2.89), the required $\sin\psi = \pi/4, \sin\psi' = \pi/4$ result in the fixed value of such parameters, therefore not compatible with the M_W anomaly. Such patterns can also be seen from Fig 3.2a, where the center $(\pi/4, \pi/4)$ is excluded from the contour.

3.3 Fitting M_W with $SU(6)/Sp(6)$ little higgs model

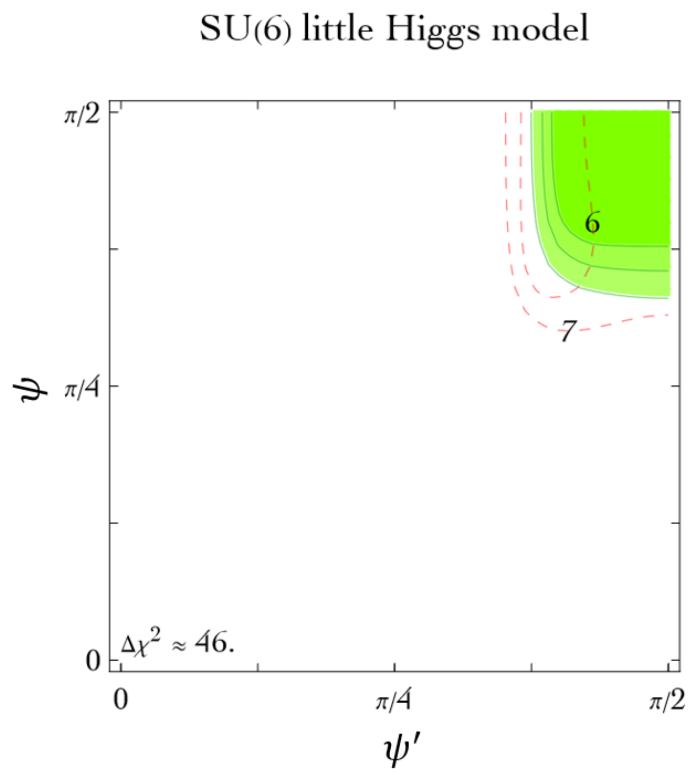
With the same method, the modified little higgs theory with $SU(6)/Sp(6)$ can also be investigated. After the implementation of Eq (2.82) and Eq (3.9), the universal corrections are now

$$\begin{aligned} \hat{S} &= \frac{2M_W^2}{g^2 f^2}\left(\cos^2\psi + \frac{2\cos^2\theta_W}{\sin^2\theta_W}\cos^2\psi'\right) \\ \hat{T} &= \frac{M_W^2}{g^2 f^2}(5 + \cos 4\beta) \\ W &= \frac{4M_W^2}{g^2 f^2}\cos^4\psi \\ Y &= \frac{8M_W^2}{g'^2 f^2}\cos^4\psi' \end{aligned} \tag{3.12}$$

where the new parameter $\tan\beta = v_2/v_1$ is the ratio of the VEVs of the two higgs doublets, as discussed in the preceding section. Under the assumption $\cos 4\beta = 0$ to simplify the calculation, which means that the two higgs VEVs will have a ratio ~ 0.414, either β or $1/\beta$, the fitting results is to indicate a descended energy scale $f \sim 7$ TeV, as shown in Fig 3.2b.



(a)



(b)

Figure 3.2: The fitted diagrams with ψ, ψ', where the red contour is the specific value of energy scale f in TeV.

3.4 The incompatibility of simplest little higgs

Though the littlest higgs and the modified $SU(6)/Sp(6)$ models show satisfactory compatibility with the recent M_W anomaly, the simplest little higgs model, on the other hand, denies the correction of the CDF discovery. The analysis of how the violation takes place shall be a little more complicated than the previous study on similar models, since the existence of an heavy extra Z' boson.

The Z' boson gives non-universal corrections to the precision observables, therefore the extracted $\hat{S}, \hat{T}, W, Y$ can not fully explain the modifications on the model. Fortunately, the incompatibility of the simplest little higgs model can be seen from the incoherence of these parameters with the M_W anomaly, which can be seen from the following derivations.

To define an extra Z' boson, the following parameters are indispensable: a Z' gauge coupling constant $g_{Z'}$, the mass term $M_{Z'}$ and the Z' charges for SM fermions and the higgs boson: $Z'_H, Z'_L, Z'_{e_R}, Z'_Q, Z'_{u_R}, Z'_{d_R}$.[19] For a universal Z', it is obviously that such charge will be identical to the SM hypercharge, $Z'_i = Y_i$. The oblique parameters are given as[18]

$$\begin{aligned}
\hat{S} &= \frac{M_W^2}{M_{Z'}^2}\left(c_W - \frac{c_Y g}{g'}\right)\left(c_W - c_Y\frac{g'}{g} - 2\frac{g_{Z'}Z'_H}{g}\right) \\
\hat{T} &= \frac{M_W^2}{M_{Z'}^2}\left[\left(c_Y\frac{g'}{g} + 2\frac{g_{Z'}Z'_H}{g}\right)^2 - c_W^2\right] \\
W &= \frac{M_W^2}{M_{Z'}^2}c_W^2 \\
Y &= \frac{M_W^2}{M_{Z'}^2}c_Y^2
\end{aligned} \tag{3.13}$$

where the coefficients c_W, c_Y are defined as

$$\begin{aligned}
c_Y &= \frac{g_{Z'}Z'_{e_R}}{g'Y_{e_R}} \\
c_W &= \frac{2g_{Z'}}{Y_{e_R}g}(Z'_{e_R}Y_L - Z'_L Y_{e_R})
\end{aligned} \tag{3.14}$$

For the expression of $\hat{T}$, the equation can also be written as

$$\hat{T} = \frac{M_W^2}{M_{Z'}^2}\left[\left(c_Y\frac{g'}{g} + 2\frac{g_{Z'}Z'_H}{g}\right)^2 - c_W^2\right] = \frac{g_{Z'}^2}{g^2}[(-Z'_{e_R} + 2Z'_H)^2 - (-Z'_{e_R} + 2Z'_L)^2] \tag{3.15}$$

where the SM hypercharge $Y_{e_R} = -1, Y_L = -1/2$ are implemented. Under the condition $Z'_H = Z'_L$, $\hat{T} = 0$ is extracted from as the equation shows.

For simplest little higgs model, the $U(1)_X$ just stands for the Z' charges, therefore the gauge coupling $g_{Z'}$ and the mass $M_{Z'}^2$ are

$$g_{Z'} = \frac{g}{\sqrt{1 - \frac{g'^2}{3g^2}}} \qquad M_{Z'}^2 = \frac{2f^2 g^2}{3 - \frac{g'^2}{g^2}} \tag{3.16}$$

which is just the implementation of the discussion in Sec 2.2.5 and the detailed derivation in Ref [9]. The corresponding oblique parameters are

$$\hat{S} = 4W = \frac{4g^2 Y}{g'^2} = \frac{2M_W^2}{f^2 g^2}, \qquad \hat{T} = 0 \tag{3.17}$$

the latter of which is a direct corollary of Eq (2.33), where the defined $X_{\Psi_L} = X_\phi = -1/3$. Such parameterization can not be in coherence with the M_W correction, since there are no correction given for $\hat{T}$ at all.

Chapter 4

Extrapolation: From the Little Higgs

In the previous sections, some promising little higgs models are introduced and fitted to the recent M_W anomaly, some of which are compatible and some not. In this chapter, we will focus on the possible phenomenological prediction of some little higgs model, as well as some interesting mechanism happening in the energy scale of the little higgs.

4.1 Lepton Flavor Violation with little higgs

The **lepton flavor violation(LFV)** process is that one heavy lepton decay into one or several leptons of different flavor, such as $\mu \to e, \tau \to \mu$, the energy scale of which shall take place at around ~ 10 TeV scale. In Ref [20, 21, 22], the researchers propose the LFV compatible with the L^2HT and simplest little higgs model, where they discussed the decay process $l \to l_a\gamma$ and $l \to l_a l_a \bar{l}_a$ with penguin diagram and box diagram, coming from the one-loop corrections.

For the simplest model, there should be 3 more heavy neutrinos enabling the LFV process, especially within the nuclei, where $\mu \to e\gamma$ or $\mu \to 3e$ take place. The constraint on energy scale f is rather loose: f is estimated from 7.5 TeV to as large as 85 TeV. Such high upper limit, to a great extent, is set to satisfy the condition for the LFV. For the exclusive parameter in simplest little higgs model $\tan\beta = f_1/f_2$, where they are defined in Eq (2.35), the fitted result is $\tan\beta \in [1, 9]$ from the large interval of total f.

For the Littlest higgs model with T parity, it is compatible with an additional inverse seesaw mechanism, also generating 3 heavy majorana neutrinos. Analysis focused on τ lepton decaying into hardons: $\tau \to \mu P$, $\tau \to \mu V$ and $\tau \to \mu P\bar{P}$, with $P(V)$ a pseudoscalar (vector) meson, namely π^0, η, η' in single meson channel while $\pi^\pm, K$ in double channel, and $V = \rho, \phi$. Again, such process are generated via the high energy as far as ~ 10 TeV scale, and by the one-loop penguin and box diagram. The conclusion they gave for the L^2HT T-odd fermions are all below 4 TeV, and

the new physics energy scale $f \sim 1.5$ TeV, which is quite different from the results for littlest higgs, where its energy scale is as large as 4 times the L^2HT one. This comes from the reason that the heavy majorana neutrinos are at ~ 19 TeV and has relation $M \sim 4\pi f$, suppressing the energy scale.
In conclusion, the little higgs compatible LFV shows great research potential for both the model itself, as well as the leptonic behavior. Such mechanisms, as the numerical value shows, shall be excavated with the LHC and future collider.

4.2 Little Higgs Phenomenology

In chapter 3 we studied the effects of several little higgs models on EWPO, via the presentation of the oblique parameters, but only a fraction of them, and gave the compatible energy level region. In Ref [23], a global fit based on experimental results for the littlest higgs model are performed to show the bound of energy level f as in Fig 4.1, where f is given as a function of $c' = \cos\psi'$. The result is in coherence with the derivation from Section 3.2, which shall corresponds to $f \sim 9$TeV at $\cos\psi' \sim 0$. Inserting the T-parity for the littlest higgs change the situation as shown in Fig 4.2, in which the lower bound at the same condition $\cos\psi' \sim 0$ reduce to $f > 3.6$TeV. However, as the previous section indicates, the non-universal effects including lepton flavor violation appears in this model. The $SU(6)/Sp(6)$ model is considered in Ref [5, 13, 18], which gives the similar result $f > 3$TeV, since the model itself is just a modification based on Littlest Higgs. Such result is also looser than the finding in Section 3.3.

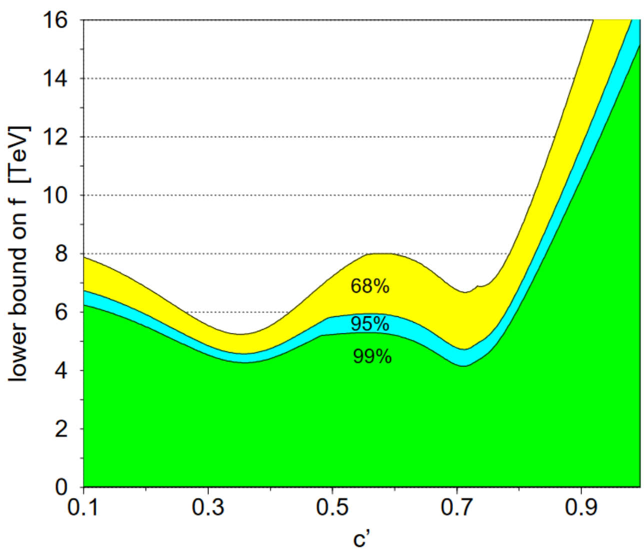


Figure 4.1: The regions of excluded f value as the function of $c' = \cos\psi'$ from the C.L 68%(yellow), 95%(blue), and 99%(green).[23]

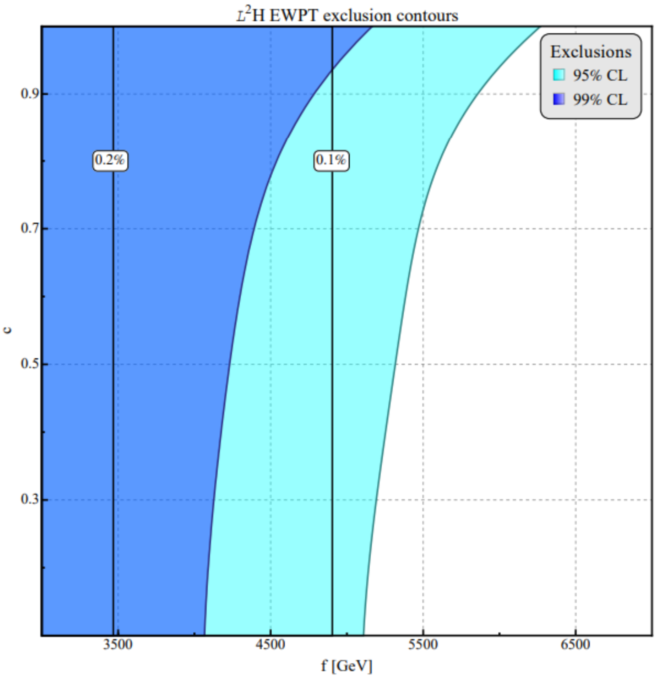


Figure 4.2: The regions of excluded f value as the function of $c' = \cos\psi'$ (reversed from Fig 4.1) from the C.L 95%(light blue), and 99%(dark blue).[24]

After a brief revisited discussion about the energy scale f, the accompanied heavy gauge bosons, the heavy quark and the potential triplet from the littlest higgs model shall be explored along with the new physics at such energy level. The gauge boson W_H discovery should be through the following channels with partial width

$$\begin{aligned}\Gamma(W_H^3 \to l^+l^-) &= \frac{g^2\cot^2\psi}{96\pi^2}M \\ \Gamma(W_H^3 \to \bar{q}q) &= \frac{g^2\cot^2\psi}{32\pi^2}M \\ \Gamma(W_H^3 \to Zh) &= \frac{g^2\cot^2 2\psi}{192\pi^2}M \\ \Gamma(W_H^3 \to W^+W^-) &= \frac{g^2\cot^2\psi}{96\pi^2}M\end{aligned} \tag{4.1}$$

with $M = M(W_H)$. The available channels are discussed in detail in Ref [11]. The first one $W_H^3 \to l^+l^-$, is the cleanest since the decay channel only generate lepton. However, such pattern is not characteristic and is not the unique consequence of the littlest higgs model. Therefore, a more ripen method is to analyse the $W_H^3 \to Zh$ channel, where the $\cot 2\psi$ is a characteristic coefficient from Eq (2.56). The ATLAS collaboration is now undertaking the measurement, with the prediction and simulation[25] of the possible channels on Fig 4.3, in which only the final product from the Zh and $W^{\pm}h$ decaying channels are listed. For the Zh one, the typical final remnants are $l^+l^-\bar{b}b$ and $jj\gamma\gamma$, and for $W^{\pm}h$, they are $l^{\pm}\nu\bar{b}b$ and $jj\gamma\gamma$.

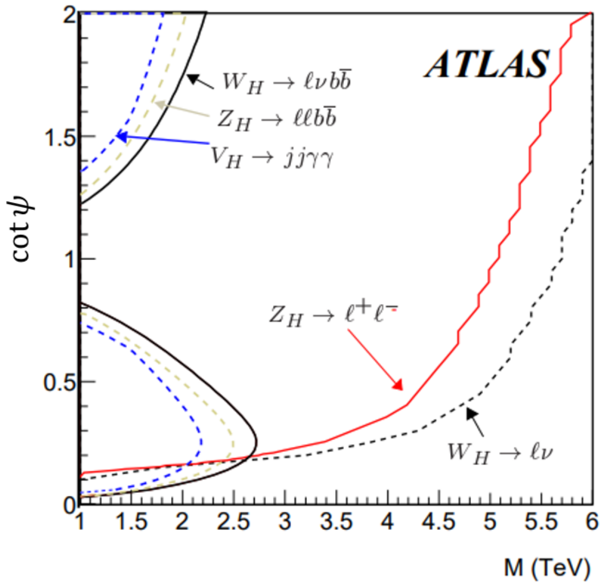


Figure 4.3: Simulated results for the accessible regions with the $\cot\psi$ versus $M(W_H)$ plot[25]

Unfortunately, the heavy top quark itself can not be the decisive evidence of the presence of the little higgs model either, since many other plausible theories provide such predictions at the TeV scale. To be confident that the T quark is induced by the littlest higgs model, the Eq (2.64) shall be emphasized. With the known λ_t from Standard Model, the M_T shall be measured from LHC and future colliders. Combined with, if possible, the observed W_H, the accurate value of f shall be determined and verify the littlest higgs model. With the recent search on the T quark along with its coupling κ_T to SM particle[26], the coupling κ_T is 0.35 when the M_T is around 1.2 TeV, and 1.6 for M_T as large as 2.3 TeV. This result shall help measure the true mass of the potential heavy top quark, and testify the little higgs model.

Chapter 5

Conclusion: The HEP awaited

In this dissertation, we introduce the latest observation of M_W anomaly, reviewing the Standard Model electroweak theory as the introduction, which inherits the gauged W boson. Then the transition from SM electroweak energy to higher energy is discussed, where we extract the idea of an energy scale interval f connecting the realistic macroscopic world to the ultraviolet *hierarchy problem* at TeV. Subsequently, the little higgs model is implemented as a competitive theory predicting the energy scale itself and the new physics with novel particles. We introduce the little higgs mechanism comprehensively, focusing on several discrete models: the simplest little higgs, the littlest higgs, the littlest higgs with T parity as a modification, and the little higgs with $SU(6)/Sp(6)$ breaking. Moreover, the compatibility of these models with the M_W anomaly is discussed in the following pages, where for the compatible littlest higgs and $SU(6)/Sp(6)$ little higgs, the calculation gives a prediction or limitation on the energy scale f. Finally, the phenomenology and possible leptonic flavor violation mechanism in the designated energy scale is argued, giving specific limitation on the energy scale of the undiscovered particles.
The limitation of the energy scale is slightly different in these models, from the lowest 1.5 TeV predicted by littlest higgs with T parity, along with the lepton flavor violation pattern forecasting a 19 TeV heavy neutrinos, to the original littlest higgs at $f > 9$ TeV. Onto the road of such high energy, there will be patterns involving heavy gauge boson, and the presence of a heavy top quark. These potential particles shall be discovered with the unique decaying channel implying the remnant of little higgs mechanism, and shall be discovered with the current frontier accelerator the High-Luminosity Large Hardon Collider (HL-LHC), additionally the possible next-generation collider such as Circular Electron Positron Collider (CEPC), Compact linear collider (CLIC) and the Future Circular Collider (FCC), with parameters described in Table 5.1.

The little higgs model provides a bridge connecting the SM energy and the UV completion. Studying the effects of such models will definitely contributes to the

Collider	HL-LHC[27]	CEPC[28]	CLIC[29]	FCC-ee[30, 31]	FCC-eh
Luminosity$(ab^{-1}year^{-1})$	3	0.4	0.5	5	2
CoM Energy(GeV_{ee})	-	240	380∼3000	240∼365	-
CoM Energy(TeV_{hc})	13	75∼125	-	-	50
Possible Build Year	Operating	$\sim$ 2030	$\sim$ 2040	$\sim$ 2036	$\sim$ 2050

Table 5.1: The basic parameters of high energy colliders under operation and on proposal. The center-of-mass colliding energy are divided into leptonic collision (GeV_{ee}) and hardonic collision (TeV_{hc}).

development of the theoretical implication of grand unification theory, supergravity and the origin of our universe. Low energy effects concerning the EWPO and higher order precision confinements shall also contribute to the development of new theories, correcting the predictions and indicating directions for them. With the recent M_W anomaly, we hope such BSM postulates will one day be testified to be correct, or to be special case for a more general unified theory.

Appendix A

Non-linear sigma model

In this appendix we will interpret what the 'non-linear' refers to in its name. Back to Section 2.1, where we use the example as a G/H condition. Considering an element $g_0 \subset H$. we can write it as

$$g_0 = h_0 \tag{A.1}$$

where the exponential term is simply $\mathbb{1}$, another general element g' is as Eq (2.9) goes

$$g' = e^{\mathrm{i}c'^l P'_l} h' \tag{A.2}$$

Then the product is

$$g_0 g' = h_0 e^{\mathrm{i}c'^l P'_l} h' \subset G \tag{A.3}$$

But we can also define a $g \subset G$ to be

$$g = e^{\mathrm{i}c^l P_l} h = g_0 g' = h_0 e^{\mathrm{i}c'^l P'_l} h' \tag{A.4}$$

Thus the elements from the subgroup and coset will have a correspondence

$$h_0 e^{\mathrm{i}c'^l P'_l} h' = e^{\mathrm{i}c^l P_l} h \tag{A.5}$$

After a product calculation within subgroup H, namely $hh'^{-1} = h_1$, Eq (A.5) now becomes

$$h_0 e^{\mathrm{i}c'^l P'_l} h_0^{-1} h_0 = e^{\mathrm{i}c^l P_l} h_1 \tag{A.6}$$

This equation implies that the $h_0 e^{\mathrm{i}c'^l P'_l} h_0^{-1} = e^{\mathrm{i}c^l P_l}$, $h_0 = h_1$ and therefore the transformation of $c^l \to c'^l$ is linear if the element g_0 is within the subgroup.
However, if g_0 is instead $g_0 \subset G/H$

$$g_0 = e^{\mathrm{i}c_0^l P_{l0}} \tag{A.7}$$

then with Eq (A.2) and Eq (A.4) we can get, finally

$$e^{\mathrm{i}c_0^l P_{l0}} e^{\mathrm{i}c'^l P'_l} = e^{\mathrm{i}c^l P_l} \tag{A.8}$$

in which the c^l is clearly transformed non-linearly. This just corresponds to the higgs field defined in Section 2.3, where they are in the coset group.

Appendix B

Coleman-Weinberg Potential

The Coleman-Weinberg effective potential[32] is useful in dealing with the radiative correction in spontaneous symmetry breaking models, which was firstly proposed by Coleman and Weinberg in 1973.

Considering a simple massless, quadratically-interacting real meson field, with Lagrangian density

$$L = \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{\lambda}{4!}\varphi^4 + \frac{1}{2}(Z_\varphi - 1)(\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{1}{2}Z_m \varphi^2 - \frac{1}{4!}(Z_\lambda - 1)\varphi^4 \quad \text{(B.1)}$$

where the term $Z_\varphi, Z_m, Z_\lambda$ are the coefficients for renormalization, and the latter 3 terms serves as the counterterm. Notice the though the example is a simple massless condition, there will still be a mass counterterm.

To the lowest order, in tree level, the only contribution to the potential is

$$V = \frac{\lambda}{4!}\varphi^4 \quad \text{(B.2)}$$

which gives a simple diagram of four external legs and one vertex. However, the next order calculation including the one-loop interaction of the terms results in all the terms and counterterms, as well as an infinite series coming from the Eq (B.2) in one-loop diagrams, contributing to the overall effective potential

$$V = \frac{\lambda}{4!}\varphi^4 + \frac{1}{2}Z_m\varphi^2 + \frac{1}{4!}(Z_\lambda - 1)\varphi^4 + \mathrm{i}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{\lambda\varphi^2}{2(k^2+\mathrm{i}\epsilon)}\right)^n \quad \text{(B.3)}$$

where the latter term is just the propagators integrated within the n vertex one-loop. Rotation of the k^0 into $\mathrm{i}k^0$ shall be implemented to convert the integral into a Euclidean one, which reads

$$V_{int} = \int_{\mathbb{R}} \frac{\mathrm{d}^4k}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{\lambda\varphi^2}{2k^2}\right)^n \quad \text{(B.4)}$$

Using the taylor expansion for $\ln(1+x) = \sum x^n/n$, the total effective potential can be derived into

$$V = \frac{\lambda}{4!}\varphi^4 + \frac{1}{2}Z_m\varphi^2 + \frac{1}{4!}(Z_\lambda - 1)\varphi^4 + \frac{1}{2}\int \frac{\mathrm{d}^4k}{(2\pi)^4} \ln\left(1 + \frac{\lambda\varphi^2}{2k^2}\right) \tag{B.5}$$

To start the renormalization process, the integral is firstly evaluated with a cutoff $k = \Lambda$, resulting

$$V = \frac{\lambda}{4!}\varphi^4 + \frac{1}{2}Z_m\varphi^2 + \frac{1}{4!}(Z_\lambda - 1)\varphi^4 + \frac{\lambda\Lambda^2}{64\pi^2}\varphi^2 + \frac{\lambda^2\varphi^4}{256\pi^2}\left(\ln\frac{\lambda\varphi^2}{2\Lambda^2} - \frac{1}{2}\right) \tag{B.6}$$

where the form of $1/\Lambda^2$ terms all go to 0 therefore excluded from the expression. To make the renormalized mass term to vanish

$$\frac{\mathrm{d}^2V}{\mathrm{d}\varphi^2}(\varphi = 0) = 0 \Rightarrow Z_m = -\frac{\lambda\Lambda^2}{32\pi^2} \tag{B.7}$$

Moreover, the quartic condition is

$$\frac{\mathrm{d}^4V}{\mathrm{d}\varphi^4}(\varphi = M) = \lambda \Rightarrow Z_\lambda - 1 = -\frac{3\lambda^2}{32\pi^2}\left(\ln\frac{\lambda M^2}{2\Lambda^2} + \frac{11}{3}\right) \tag{B.8}$$

with M satisfying $Z_\varphi(M) = 2$, and the corresponding total effective potential reads

$$V = \frac{\lambda}{4!}\varphi^4 + +\frac{\lambda^2\varphi^4}{256\pi^2}\left(\ln\frac{\varphi^2}{M^2} - \frac{25}{6}\right) \tag{B.9}$$

This is the idea about the generation of a simple massless meson's Coleman Weinberg Potential. For a more complicated case like discussed in Section 2, the non-abelian Coleman Weinberg potential V is consisted from $V = V_0 + V_2 + \cdots$ where

$$V_2 = V_s + V_g + V_f \tag{B.10}$$

which are the quadratic contributions from respectively scalar, gauge-fields, and fermions, along with a zero order V_0. These contributions can be calculated separately by

$$\begin{aligned} V_s &= \frac{1}{64\pi^2}\mathrm{Tr}[W^2 \ln W], \qquad W_{ab} = \frac{\partial^2 V_0}{\partial\varphi_a\partial\varphi_b} \\ V_g &= \frac{3}{64\pi^2}\mathrm{Tr}[M^4 \ln M^2], \qquad M^2_{ab} = \frac{\partial^2 L}{\partial A_{\mu a}\partial A^\mu_b} \\ V_f &= -\frac{1}{64\pi^2}\mathrm{Tr}[(mm^\dagger)^2 \ln(mm^\dagger)], \qquad m_{ab} = y_{ab} \end{aligned} \tag{B.11}$$

in which the y_{ab} is simply the yukawa coupling term for fermion term $y_{ab}\bar{\psi}_a\psi_b$. These terms are useful in dealing with the quadratically divergent terms in little higgs models.

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